Math AA HL Chapter 4 Polynomial functions

Polynomial $f(x) = ax^n + bx^{n-1} + ... + px + c$ all exponents are positive integers.
Degree is the highest exponent power of x or highest exponent of the variable.
Leading coefficient – coefficient of the term with the highest exponent.

Linear – 1st degree Quadratic – 2nd degree Cubic – 3rd degree
Quartic – 4th degree Quintic – 5th degree

Value of a polynomial –

Direct substitution – substitute the value of x into the polynomial and calculate.

Nested scheme – synthetic substitution – make a chart with the coefficients then add down and multiply diagonally. The value of the polynomial when the value is substituted is the last number. Remember to enter zeros when powers of the variable are missing.

The Remainder Theorem: If a polynomial *f*(x) is divided by (x – h) the remainder is *f*(h).
The Factor Theorem: If *f*(h) = 0 then (x – h) is a factor of *f*(x).

Possible roots are the factors of the constant over the factors of the leading coefficient.
For the function $f(x) = ax^n + bx^{n-1} + ... + px + c$ possible $\pm\dfrac{\text{factors of } c}{\text{factors of } a}$

Sum and Product of roots: sum of roots $\dfrac{-b}{a}$ product of roots $\dfrac{c}{a}(-1)^n$

AHL 2.12	Sum and product of the roots of polynomial equations of the form $\sum_{r=0}^{n} a_r x^r = 0$	Sum is $\dfrac{-a_{n-1}}{a_n}$; product is $\dfrac{(-1)^n a_0}{a_n}$

Graphs: Remember that the roots are the values where the graph intersects the x-axis. If the graph is tangent to the x-axis then there will be a double root at that value.

Long division must be used to divide a polynomial by a polynomial that is not linear.
Synthetic division can only be used to divide when the divisor is linear. (then make sure the divisor has a leading coefficient of 1)
Synthetic substitution can be used to find the remainder. Given f(x) and x – a the value of f(a) will be the remainder when f(x) is divided by x – a.

Practice: Divide.

Ex. 1 $\dfrac{2x^3 - 7x^2 + 6x - 4}{x - 3}$ Ex. 2 $\dfrac{3x^4 - 4x^3 + 5x^2 - 7x + 4}{x^2 + 2}$