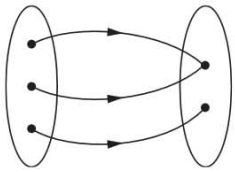
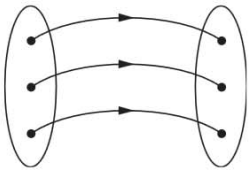
IB Math AA HL Chapter 3 Functions Notes

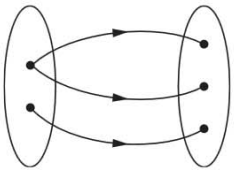
A function is a mathematical rule. Although the word "function" is often used for any mathematical rule, this is not strictly correct. For a mathematical rule to be a function, each value of x can have only one image.



Many to one
(a function)



One to one
(a function which has an inverse)



One to many
(not a function)

Vocabulary:
Domain – the set of numbers that provide the input for the rule.
Image – the output from the rule of an element in the domain.
Range – the set of numbers consisting of the images of the domain.
Co-domain – a set containing the range.
Function – a rule that links each member of the domain to exactly one member of the range.

Two types of notations for functions exist, they are f(x) = ax + b or f:x → ax + b

Review the different types of numbers.

$\mathbb{Z}$ – the set of integers $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
$\mathbb{Z}^+$ – the set of positive integers $\{1, 2, 3, \ldots\}$
$\mathbb{N}$ – the set of natural numbers $\{0, 1, 2, 3, \ldots\}$
$\mathbb{Q}$ – the set of rational numbers $\left\{x : x = \frac{p}{q}, p, q \in \mathbb{Z}\ q \neq 0\right\}$
$\mathbb{R}$ – the set of real numbers.

Composite functions: one function followed by another $f(g(x))$ or $f \circ g(x)$

The order is important $f(g(x)) \neq g(f(x))$ accept in rare cases.

Inverse functions: An inverse function is the reverse of the function. It allows one to find the input when they start with the output. $f^{-1}(x)$ means the inverse of $f(x)$

How to find an inverse:

1. Check that an inverse function exists for the given domain.
2. Rearrange the function so that the subject is x.
3. Interchange x and y.

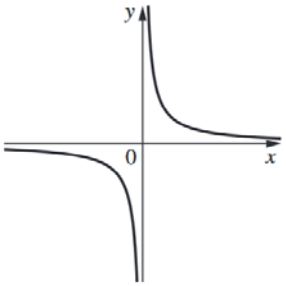
The graph of inverse functions.

The graph is the reflection over the y = x line.
If a point on the graph is (x, y) then the point on the inverse is (y, x).

Reciprocal functions

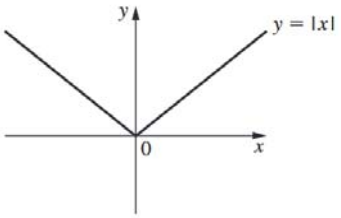
$$f(x)=\frac{1}{x}$$

Vertical asymptotes
$x \neq 0$ or denominator $\neq 0$



Absolute value function or piecewise function

$$f(x)=|x| \qquad \text{or} \qquad f(x)=\begin{cases} x, & x \geq 0 \\ -x, & x<0 \end{cases}$$



Drawing a graph:

Roots – value of x when y = 0
y-intercept – the value of y when x = 0
turning points – vertex or where the graph changes direction
vertical asymptotes – when y is not defined
horizontal asymptotes – when $x \rightarrow \pm\infty$

Transformations:
For ***kf(x)***, each y-value is multiplied by k and so this creates a vertical stretch.
For ***f(kx)***, each x-value is multiplied by k and so this creates a horizontal stretch.
For $\boldsymbol{f(x)+k}$, $\boldsymbol{k}$ is added to each y-value and so the graph is shifted vertically.
For $\boldsymbol{f(x+k)}$, $\boldsymbol{k}$ is added to each x-value and so the graph is shifted horizontally.
For $\boldsymbol{-f(x)}$, each y-value is multiplied by -1 and so each point is reflected in the x-axis.
For $\boldsymbol{f(-x)}$, each x-value is multiplied by -1 and so each point in reflected in the y-axis.

Rational Functions $\boldsymbol{f(x)=\frac{g(x)}{h(x)}}$ specifically $\boldsymbol{f(x)=\frac{a}{px+q}}$ and $\boldsymbol{f(x)=\frac{bx+c}{px+q}}$

$\boldsymbol{f(x)=\frac{a}{px+q}}$ this is a transformation of a reciprocal function $\boldsymbol{f(x)=\frac{1}{x}}$

$\boldsymbol{f(x)=\frac{bx+c}{px+q}}$ this is similar to $\boldsymbol{f(x)=\frac{a}{px+q}}$ but it has a horizontal asymptotes.

Horizontal asymptotes $y=\frac{b}{p}$ as when $x \rightarrow \pm\infty$, $y=\frac{b}{p}$

Even and Odd functions:
A function is odd. if for all x in the domain, f(-x) = -f(x). Symmetric about the origin
A function is even, if for all x in the domain, f(-x) = f(x). Symmetric across the y-axis