Math AA HL Chapter 22  Continuous Probability Distributions

Continuous Random Variables:  The probability is equal to the area under the curve so to find the probability we integrate the function and then evaluate it over an interval.

For the range $a \le x \le b$  $\int_a^b f(x)\,dx = 1$

Expectation  E(x) =  $E(x) = \int_a^b x\, f(x)\,dx$ for values $a \le x \le b$

If probability density functions are symmetric then E(x) is the line of symmetry.  This is true for Normal distribution.

Expectation for g(x)  $E(g(X)) = \int_a^b g(x) f(x)\,dx$ for values $a \le x \le b$

Expectation for $X^2$   $E(X^2) = \int_a^b x^2 f(x)\,dx$ for values $a \le x \le b$

Variance:  Var(X) = $E(X - \mu)^2$  or  Var(X) = $E(X^2) - E^2(X)$  so

$$\text{Var(X)} = \int_a^b x^2 f(x)\,dx - \left(\int_a^b x f(x)\,dx\right)^2$$

$$\sigma^2 = \text{Var(X)} = \int_a^b (x-\mu)^2 f(x)\,dx$$

The mode:  Most often – so it is determined by the maximum.  To find the maximum take the derivative of the function and find when the derivative equals zero.

Median (m) middle - $\int_a^m f(x)\,dx = \frac{1}{2}$

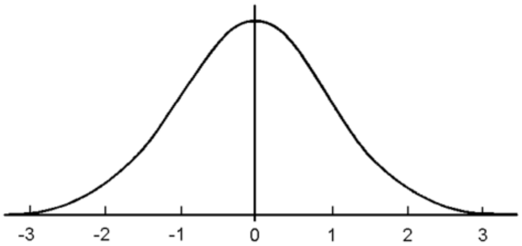
Normal Distribution

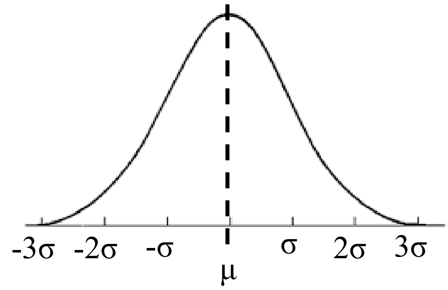
Probability density function for normal distribution  $f(x) = \frac{e^{\frac{-(x-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}$

1. area under the curve is 1
2. symmetrical around $\mu$  P(-a $\le$ X $\le$ a) = 2P( 0 $\le$ X $\le$ a) and P(X $\ge \mu$) = P(X $\le \mu$) = 0.5
3. can find probability of any value but the farther from $\mu$ the small probability.
4. approximately 95% of the values are within 2 standard deviations.
5. approximately 99.8% of the values are within 3 standard deviations.
6. the maximum value occurs when x = $\mu$ and is given by  $f(x) = \frac{1}{\sigma\sqrt{2\pi}}$
7. E(X) = $\mu$
8. Var(X) = $\sigma^2$

Calculator Standard Normal Distribution versus Normal Distribution

Standard Normal Distribution | Normal Distribution




| | |
|---|---|
| $Z \sim N(0, 1)$ | $X \sim N(\mu, \sigma^2)$ |
| lower tail | lower tail |
| $P(Z \le x)$ normalcdf($-1\times10^{99}$, x) | $P(X \le x)$ normalcdf($-1\times10^{99}$, x, μ, σ) |
| upper tail | upper tail |
| $P(Z \ge x)$ normalcdf(x, $1\times10^{99}$) | $P(X \ge x)$ normalcdf(x, $1 \times 10^{99}$, μ, σ) |

InvNorm – used to find the inverse of normal distribution. Inverse Noramal is only used with the lower tail. So upper tail must use 1- lower tail %

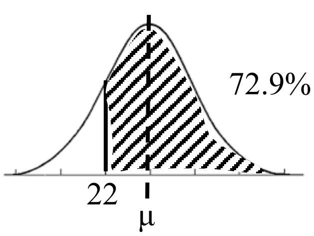
| | |
|---|---|
| $P(Z \le a) = \%$ | $P(X \le a) = \%$ |
| invNorm( % ) | invNorm(%, μ, σ) |
| $P(Z \ge a) = \%$ | $P(X \ge a) = \%$ |
| invNorm( 1-% ) | invNorm( 1-%, μ, σ) |

Finding mean (μ) or standard deviation (σ)

Must use Standard normal distribution because μ or σ are unknown. $Z = \frac{X-\mu}{\sigma}$

Example: If $X \sim N(\mu, 7)$ and $P(x \ge 22) = .729$, find the value of $\mu$.

1st make a sketch




upper tail .729 lower tail .271

invNorm(.271) = -.6097913937 ≈ -.610

$Z = \frac{X-\mu}{\sigma}$  $-.610 = \frac{22-\mu}{\sqrt{7}}$ so $\mu = 26.3$

In applications of normal distribution it is helpful to convert the given information into the symbolic equivalent.