Math AA HL Chapter 13 Vectors, Lines and Planes

Vector equation of a straight line $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \lambda \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

a is a position vector of a point on the line and **b** is the directional vector of the line.

Parametric equation of a straight line is when the vector equation is expressed in terms of λ.

$$x = a_1 + \lambda b_1 \ , \quad y = a_2 + \lambda b_2 \ , \quad z = a_3 + \lambda b_3$$

Cartesian equation of a straight line

$\dfrac{x - x_0}{l} = \dfrac{y - y_0}{m} = \dfrac{z - z_0}{n}$ where $(x_0,\ y_0,\ z_0)$ is a point on the line and $\begin{pmatrix} l \\ m \\ n \end{pmatrix}$ is the directional vector of the line.

Parallel, intersecting and skew lines.

Parallel lines have the directional vectors that are equal or multiples of each other.

To see if the lines are the same, check to see if the point from one line satisfies the other line. If a point is on both lines the lines coincide.

Intersecting and skew lines will not have directional vectors that are the same or multiples of each other. So check the directional vectors first. Secondly, check for a point of intersection. Intersecting lines will have a point of intersection but skew lines will not.

<u>Finding the intersection of two lines. p. 348</u>

1. Check the vectors are not parallel and put each line in parametric form. Make sure the parameters for each line are different.
2. Assume that they intersect, and equate the x-values, the y-values and the z-values.
3. Solve a pair of equations to calculate the values of the parameter.
4. Now substitute into the third equation. If the parameters fit, the lines intersect and if they do not the lines are skew. To find the coordinates of intersection, substitute either of the calculated parameters into the parametric equations.

<u>Finding the angle between two lines. Page 351</u>

1. Find the direction vector of each line.
2. Apply the scalar product rule and find the angle.

Equation of a plane:

Vector form: $\mathbf{r} \cdot \hat{\mathbf{n}} = d$ is the standard equation of a plane, where **r** is the position vector of any point on the plane, $\hat{\mathbf{n}}$ is the unit vector perpendicular to the plane, and d is the distance of the plane from the origin.

Cartesian form: ax + by + cz = d

Example for converting:

If $\mathbf{r}\cdot(2\mathbf{i}-4\mathbf{j}-\mathbf{k})=6$, then $(x\mathbf{i}+y\mathbf{j}+z\mathbf{k})\cdot(2\mathbf{i}-4\mathbf{j}-\mathbf{k})=6$ so 2x – 4y –z = 6

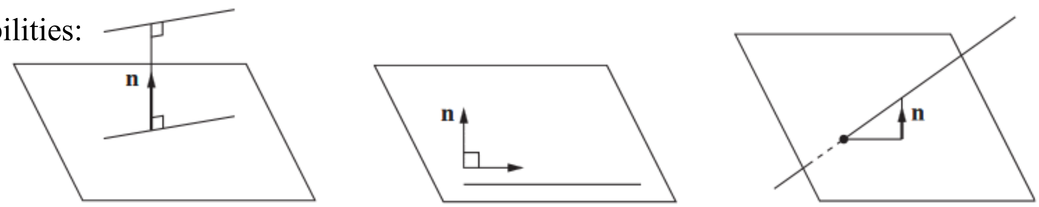
Parametric form: $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$, where **a** is the position vector of a point and **b** and **c** are direction vectors of lines, is the parametric form of the equation of a plane.

Method for converting the parametric form to scalar product form.

1. Using the direction vectors of the lines, find the perpendicular vector using the vector product. This gives the plane in the form $\mathbf{r}\cdot\mathbf{n} = D$ (notice **n** is not the unit vector, so D is not d)
2. To find D, substitute in the coordinates of the point.

Lines and Planes

Three possibilities:



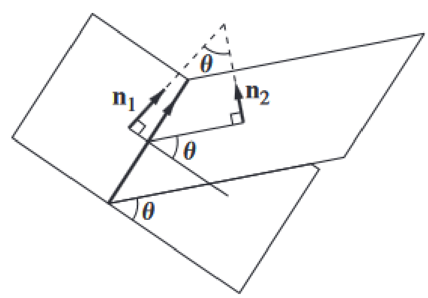
No solution Infinite solutions One solution

Finding the intersection of a line and a plane using vectors.

1. Find an expression for any point on the line, which must fit the equation of the plane.
2. Find the value of the parameter, say λ
3. Hence find the coordinates of the point of intersection.

Finding the intersection of two or three planes.

1. Find the vector product of the direction normals. This gives the direction vector of the line.
2. Write the equations of the planes in Cartesian form.
3. We now assume that z = 0 since the line has to intersect this plane.
4. Solving simultaneously gives a point on the line.
5. Write down a vector equation of the line.



Find the angle between two planes

Apply the scalar product (dot product) to the normals of the plane and find the angle.

Finding the angle between a line and a plane.

1. Use the scalar product (dot product) to find the angle between the direction normal of the plane and the direction vector of the line.
2. Subtract the angle in step 1 from 90° to find the angle between the line and plane.