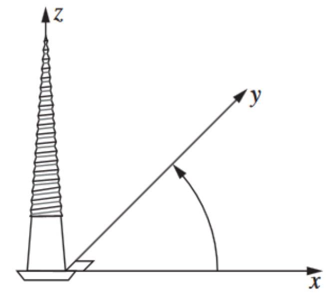
Math AA HL Chapter 12 Vectors, Lines and Planes

Scalar quantities, often called scalars, which have magnitude, but no associated direction

Vector quantities, often called vectors, which have a magnitude and an associated direction.

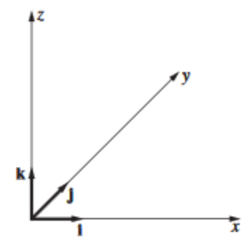
Column Vector Notation

Two dimensions $\begin{pmatrix} x \\ y \end{pmatrix}$ Three dimensions $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$



Right-hand screw rule – if a screw is placed at the origin and turned to the right the positive x axis would turn into the positive y axis.

Unit vector notation – another way to write a vector is to use the unit notation.
i is the unit vector in the x direction, **j** is the unit vector in the y direction and **k** is the unit vector in the z direction.



So a column vector of $\begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix}$ would be 2**i** + 3**j** – 2**k.**

Position Vector is the position relative to the origin. $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

Free Vector can be anywhere in space. $\mathbf{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

Tied Vector is a specific vector from one point to another, like from A to B $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

To form a tied vector subtract the A coordinates from the B coordinates (b – a for each coordinate). Also $\overrightarrow{AB} = -\overrightarrow{BA}$ because they go in the opposite direction.

Magnitude (modulus) is the length of the line representing the vector.

2 dimensions $|\mathbf{a}| = \sqrt{x^2 + y^2}$ 3 dimensions $|\boldsymbol{v}| = \sqrt{v_1^{\,2} + v_2^{\,2} + v_3^{\,2}}$, where $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$

Multiplying a vector by a scalar. If k is the scalar, to multiply by a scalar multiply each component by the scalar.

Equal vectors have the same direction and magnitude.

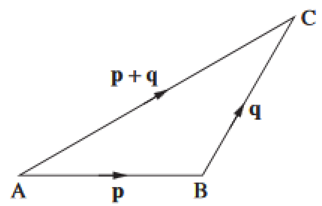
Negative vectors have the opposite direction and the same magnitude. Ex: $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ & $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

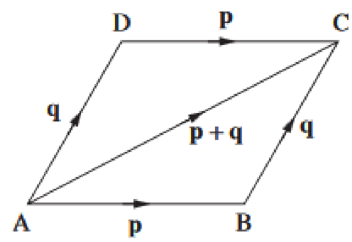
Zero vector $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ or 0**i** + 0**j** or 0**i** + 0**j** –+0**k.** Adding a vector with its negative equal 0.

Unit vectors have a magnitude of 1. If **n** is the vector then $\hat{\mathbf{n}}$ is the unit vector. To find the unit vector divide each component by the magnitude.

Parallel vectors: Parallel vectors have the same direction but are not necessarily equal. They may be scalar multiples of each other.

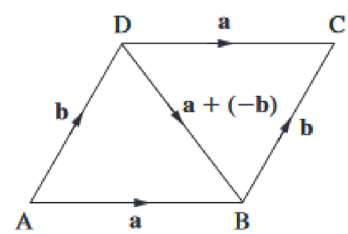
Adding vectors: Add the like components together. Geometrically this is what the addition of two vectors represents. It can be shown as a triangle or a parallelogram.





Remember vectors have direction so you must follow the direction of the arrow and if you go the other direction use the negative vector.

Subtraction is similar to addition. Algebraically just subtract the like components. Geometrically the difference of two vectors is the other diagonal of the parallelogram.
It is also helpful to think of subtraction as adding the vector from the opposite direction.



Multiplication of vectors.

Scalar Product or dot product - force

Vector Product or cross product – displacement

Scalar product $v \cdot w = |v||w|\cos\theta,$ where θ is the angle between v and w or

$$v \cdot w = v_1w_1 + v_2w_2 + v_3w_3, \text{ where } v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \ w = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$$

To find the angle between the two vectors: $\cos\theta = \dfrac{v_1w_1 + v_2w_2 + v_3w_3}{|v||w|}$

Parallel vectors: If two vectors are parallel then $a \cdot b = |a||b|\cos 0$ or $a \cdot b = |a||b|\cos\pi$

So $a \cdot b = |a||b|$

Perpendicular vectors: If two vectors are perpendicular, then $a \cdot b = |a||b|\cos\dfrac{\pi}{2}$ so $a \cdot b = 0$.

Dot Product (Scalar Product) is commutative (you can change the order) and it is distributive across addition.

Vector product $v \times w = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix}$, where $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$, $w = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$

$|v \times w| = |v||w| \sin\theta,$ where θ is the angle between v and w

Cross product is a vector perpendicular to the two vectors at their initial point. Its direction follows the right hand screw rule.

The magnitude of the cross product gives the area of the parallelogram formed by the two vectors.

Parallel vectors: If two vectors are parallel, then $\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin 0\,\hat{\mathbf{n}}$ or $\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin \pi\,\hat{\mathbf{n}}$,

So if the vectors are parallel then $\mathbf{a}\times\mathbf{b} = 0$ ($\hat{\mathbf{n}}$ is a vector perpendicular to **a** and **b**)

Perpendicular vectors: If two vectors are perpendicular, then $\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin \frac{\pi}{2}\,\hat{\mathbf{n}}$,

So if the vectors are perpendicular then $\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\hat{\mathbf{n}}$

The vector product (cross product) of two vectors is NOT commutative. They are actually vectors with opposite directions.

The vector product (cross product) can be distributed over addition. $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$

The easy way:

List vectors horizontally two times $\begin{array}{cccccc} v_1 & v_2 & v_3 & v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 & w_1 & w_2 & w_3 \end{array}$

Multiply down to the right and subtract the product from multiplying down to the left.

Ex. $v_2 w_3 - v_3 w_2$ is the first component of the cross product.

(the first cross is the first component, the second cross is the second component and the third cross is the third component.)

Example: Determine $u \times v$ *if* $u = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ *and* $v = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$. Check your answer $u \times v = \begin{pmatrix} -7 \\ -9 \\ 5 \end{pmatrix}$