Chapter 6 Sequences, Series and Binomial Theorem

Arithmetic Sequences (arithmetic progression): each term is always separated by the same amount. That amount is called the **common difference** and is denoted by *d*.

Implicit expression: the relationship between the terms is given with respect to the pervious term.

$$u_n = u_{n-1} + d$$

Explicit expression: the relationship is given with respect to the term number, *n*.

$$u_n = u_1 + (n-1)d \quad \text{sometimes} \quad u_n = a + (n-1)d$$

Arithmetic series is the sum of an arithmetic sequence.

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] \quad \text{or} \quad S_n = \frac{n}{2}\left[u_1 + u_n\right]$$

in English the sum of the first and the last terms times the number of terms divided by 2

Geometric sequence: Each term is multiplied by a common constant. This constant is known as the **common ratio**, *r*.

$$\frac{u_{n+1}}{u_n} = r \qquad u_n = ar^{n-1}$$

Sum of a **Geometric Series**: $S_n = \dfrac{a\left(r^n - 1\right)}{r-1}$ or $S_n = \dfrac{a\left(1 - r^n\right)}{1-r}$

Convergent – the gap between the terms narrows

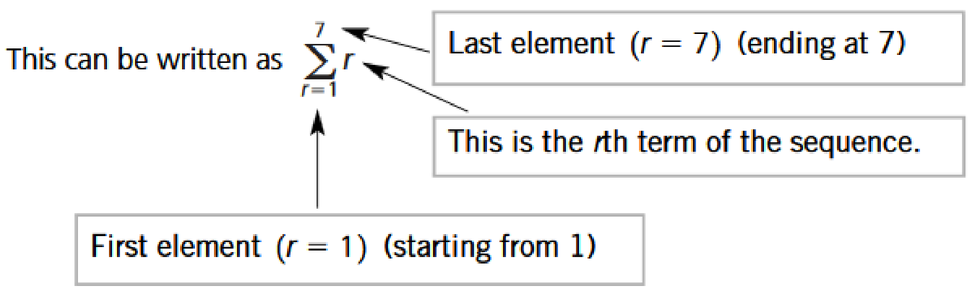
A series will only converge if |r| < 1.

Divergent – the gap between the terms widens.

All arithmetic series are divergent.

Sum to infinity is $S_\infty = \dfrac{a}{1-r} \quad \left(|r| < 1\right)$

Sigma Notation: Consider 1 + 2 + 3 + 4 + 5 + 6 + 7



Situations to remember: $\sum_{r=1}^{n} 1 = 1 + 1 + \ldots + 1 = n$ or $\sum_{r=1}^{n} a = a + a + \ldots + a = an$ where *a* is a constant.

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2} \quad \text{and} \quad \sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

Remember $\sum_{r=1}^{n} ar = a\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n}\left(ar^2 + br + c\right) = \sum_{r=1}^{n} ar^2 + \sum_{r=1}^{n} br + \sum_{r=1}^{n} c$

Remember Factorials $n! = 1 \times 2 \times 3 \times \ldots \times (n-1) \times n$

Permutations and combinations

Combinations (choose) – the order does not matter.

$^{n}C_{r}$ or $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ choosing *r* objects from a group of *n*

Permutations (pick) – the order that you pick is important.

$^{n}P_{r} = \frac{n!}{(n-r)!}$ picking *r* objects from a group of *n* (order makes a difference)

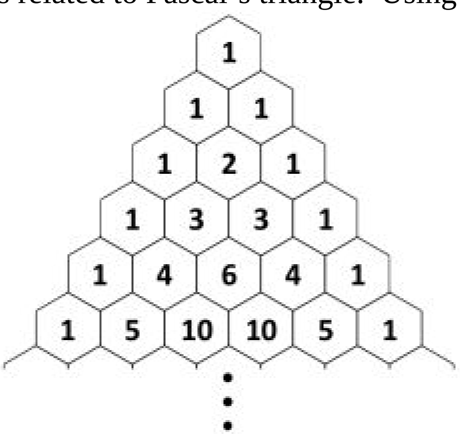
NOTE: $\binom{n}{0} = \binom{n}{n} = 1$ because $\frac{n!}{0!(n-0)!} = \frac{n!}{n!(n-n)!} = 1$

Other relationships $\binom{n}{1} = \binom{n}{n-1} = n$ and $\binom{n}{r} = \binom{n}{n-r}$ and $\binom{n}{r-1} + \binom{n}{r} = \binom{n+1}{r}$

Binomial Theorem:

$$(x+y)^{n} = \sum_{r=0}^{n} \binom{n}{r} x^{n-r} y^{r}$$ a special case is $$(1+x)^{n} = 1 + nx + \frac{n(n+1)}{2!}x^{2} + \frac{n(n+1)(n+2)}{3!}x^{3} + \ldots$$

This is related to Pascal's triangle. Using the triangle can help you find the coefficient quicker.



Later we will study the binomial expansion with rational (fractional) and integral (possibly negative) exponents.