Chapter 5 Exponential and Logarithmic Functions

Exponential Function $f(x) = a^x$

Exponent Rules: Multiply $a^m \times a^n = a^{m+n}$

Divide $\dfrac{a^m}{a^n} = a^{m-n}$

Power to a power $(a^m)^n = a^{mn}$

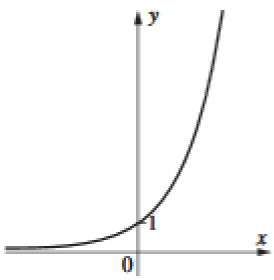
Negative exponents $a^{-m} = \dfrac{1}{a^m}$

Rational exponents $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

Zero exponent $a^0 = 1$

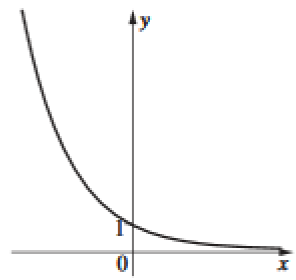
Exponential Growth

$y = a^x; \quad a > 1$



Exponential Decay

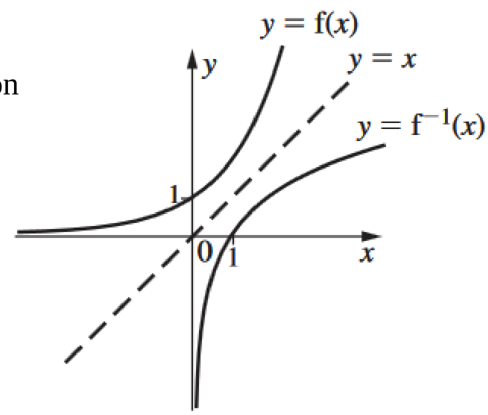
$y = a^x; \quad 0 < a < 1$ or $y = a^{-x}; \quad a < 1$



Exponential Function

Domain $\mathbb{R}$

Range y > 0



Inverse Exponential Function

Logarithm functions

Domain x > 0

Range $\mathbb{R}$

Logarithm

$y = a^x \Leftrightarrow x = \log_a y$

Compound interest

$FV = PV \times \left(1 + \dfrac{r}{100k}\right)^{kn}$, where FV is the future value,

PV is the present value, n is the number of years,

k is the number of compounding periods per year,

$r\%$ is the nominal annual rate of interest

Logarithm Facts

$\log_a 1 = 0$ and $\log_a a = 1$

$\log_a xy = \log_a x + \log_a y$

$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

$\log_a x^m = m \log_a x$

Natural log $\log_e x = \ln x$

$\ln 1 = 0$ and $\ln e = 1$

$\ln xy = \ln x + \ln y$

$\ln \left(\frac{x}{y}\right) = \ln x - \ln y$

$\ln x^m = m \ln x$

Change of base: When ever you need to you can change the base of logarithm.

$$\log_a x = \frac{\log_b x}{\log_b a} \qquad \log_a x = \frac{\ln x}{\ln a}$$

Additional Logarithm relationship: $\log_x y = \frac{\log_y y}{\log_y x} = \frac{1}{\log_y x}$

Solving exponential equations:

1. Take the natural log of both sides (or log).
2. “Bring down” the powers.
3. Divide the logs.
4. Solve for x.