Math AA HL Chapter 1 Trigonometry – Notes

Radian – One radian is an angle whose arc length is equal to the radius.

The circumference of a circle is 2π r so there are 2π radians in a complete circle or one revolution.

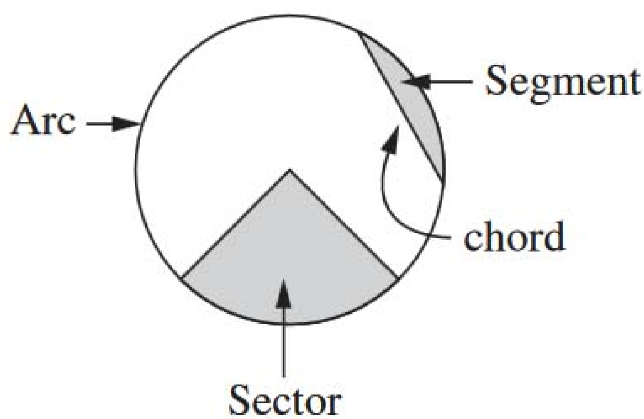
For degrees there are 360° in a circle.

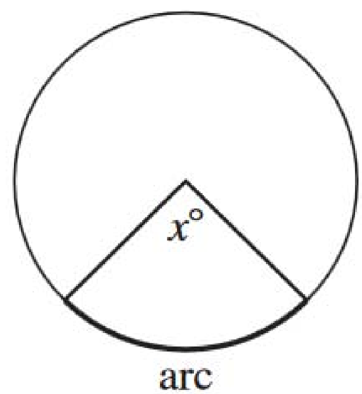
Converting Radians to Degrees or Degree to Radian: Set up a proportion comparing radians and degrees.

$2\pi \ radians = 360^\circ$ or $\pi \ radians = 180^\circ$ to convert radians to degree multiply by $\frac{180^\circ}{\pi}$

To convert degrees to radians multiply by $\frac{\pi}{180^\circ}$ Remember: the units you want go on top.

Sector of a circle is the area bounded by two radii and the arc of a circle.
An **arc** of a circle is part of the circumference of a circle.
Segment of a circle is the area between the chord of a circle and its intercepted arc.





Arc length $l = r\theta$ or $s = r\theta$ from Math 3

Area of a sector $A = \frac{1}{2}\theta r^2$ r is the radius of the circle and

θ is the measure of the central angle in radians

Area of a triangle $A = \frac{1}{2}bh$ or $A = \frac{1}{2}ab\sin C$ *a* and *b* are two sides of the triangle

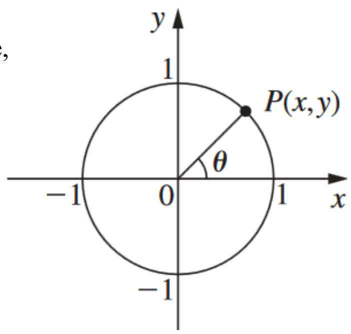
C is the included angle for sides *a* and *b*

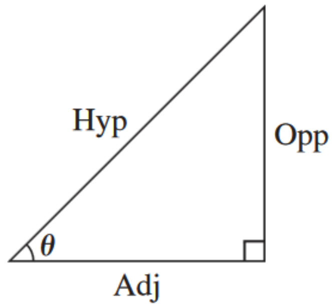
Area of a segment of a circle: Area of the sector minus the area of the corresponding triangle.

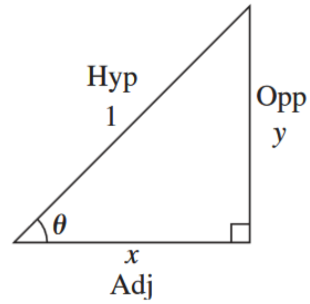
$$A = \frac{1}{2}\theta r^2 - \frac{1}{2}ab\sin\theta \quad \text{or} \quad A = \frac{1}{2}\theta r^2 - \frac{1}{2}r^2\sin\theta$$

Trigonometric Ratios or Trigonometric Functions

On a unit circle, any point P is P(cosθ, sinθ)







$$\sin\theta = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{y}{r}$$

$$\cos\theta = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{x}{r}$$

$$\tan\theta = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{y}{x}$$

$$\cot\theta = \frac{\text{adjacent leg}}{\text{opposite leg}} = \frac{x}{y}$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent leg}} = \frac{r}{x}$$

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite leg}} = \frac{r}{y}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

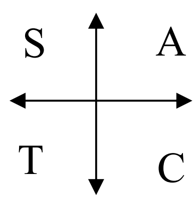
$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

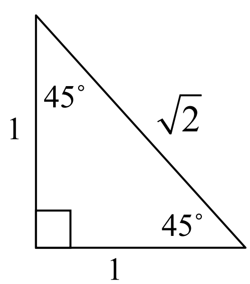
$$\sec\theta = \frac{1}{\cos\theta}$$

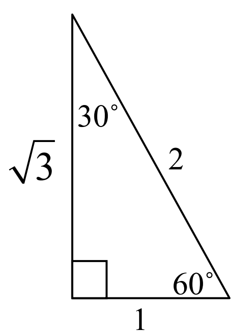
$$\csc\theta = \frac{1}{\sin\theta}$$

Remember: All Students Take Calculus or ASTC.



Special triangles





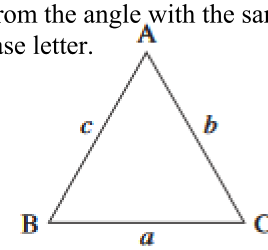
θ	0°	30°	45°	60°	90°	180°	270°	360°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	−1	0
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	−1	0	1
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined	0	undefined	0

Law of Sine $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ or $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Remember: sin A = sin(180-A) so always check for the ambiguous case.

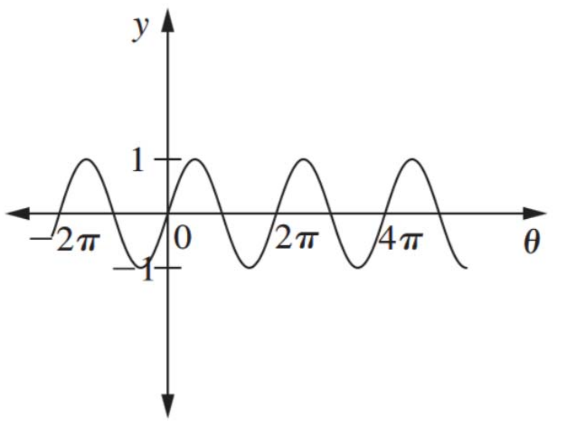
Law of Cosine $c^2 = a^2 + b^2 - 2ab\cos C$ additionally, $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

Remember: It is standard when a triangle is created to name the angles with capital letters and the side across from the angle with the same lower case letter.

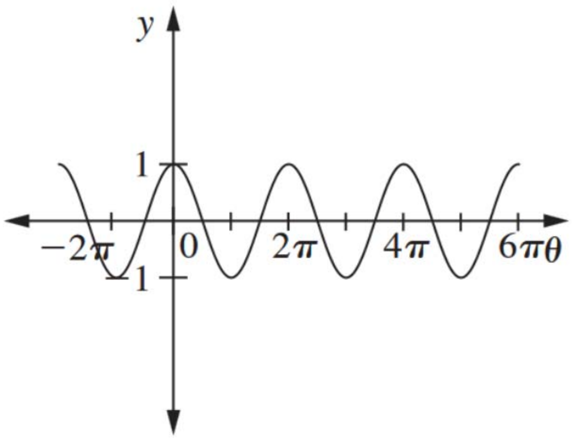


Graphs

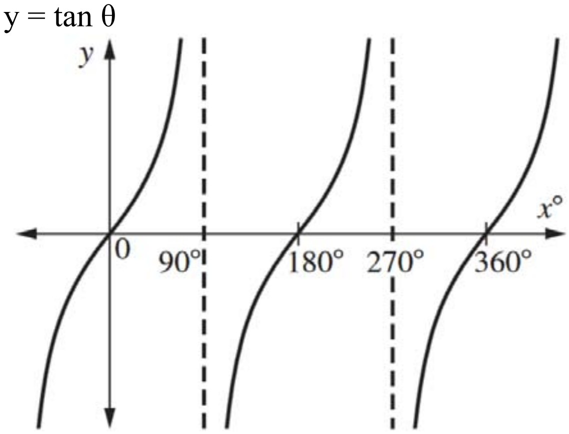
y = sin θ



y = cos θ

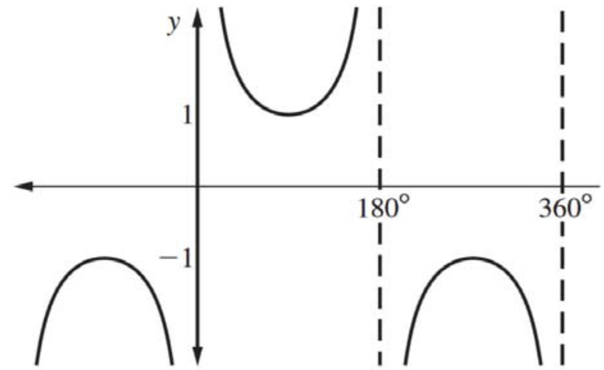


y = tan θ

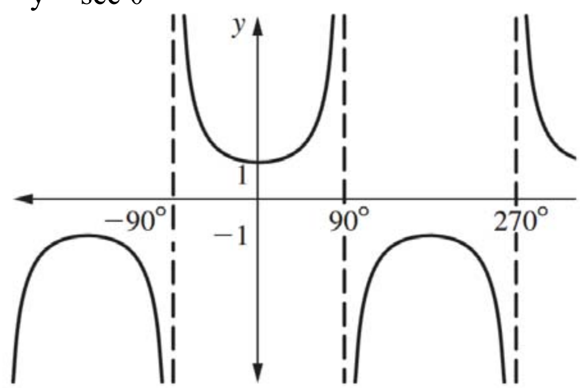


Reciprocals $\frac{1}{x}$

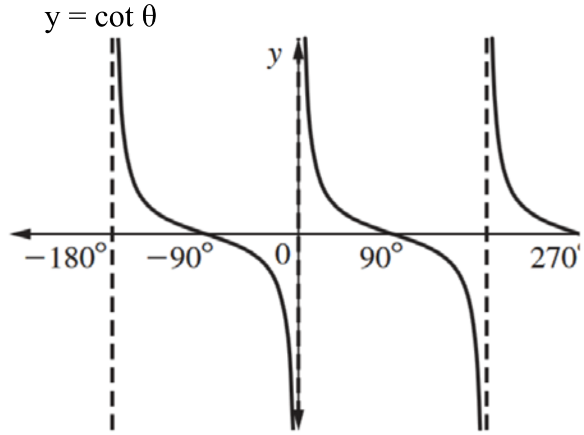
y = csc θ



y = sec θ



y = cot θ



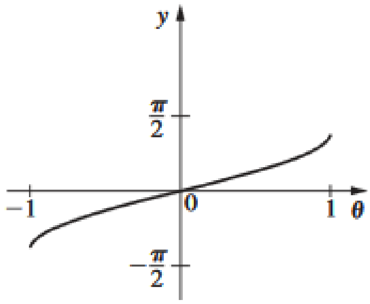
Periodic functions - Trigonometric functions are periodic because they are circular functions so they repeat every revolution (360° or 2π radians). However, tangent and cotangent repeat every 180° or π radians.

These can also be translated and dilated to give a variety of different looking graphs.

To solve equations with trigonometric functions, first solve for the trigonometric function and then take the inverse of the ratio. In order to solve with out a calculator the angle must be 30°, 45°, 60° or a multiple of them.

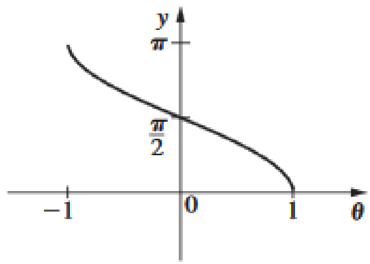
Graphing the function could also be helpful when solving an equation. Enter the equation of each side and then find the intersection.

Inverse trigonometric functions: The inverses of sine, cosine and tangent are not functions unless the domain is restricted.



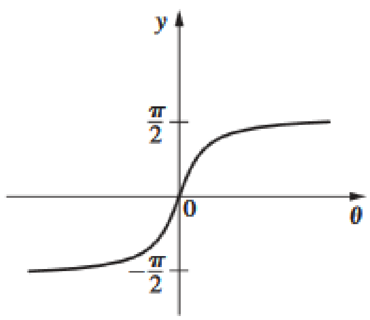
$y = \arcsin \theta$

arcsin θ is the same as $\sin^{-1}\theta$



$y = \arccos \theta$

arccos θ is the same as $\cos^{-1}\theta$



$y = \arctan \theta$

arctan θ is the same as $\tan^{-1}\theta$