

Introduction to Quadratic Functions Summary

KEY TERMS

- parabola
- vertical motion model
- roots
- second differences
- general form of a quadratic function
- factored form
- vertex
- axis of symmetry
- vertex form

LESSON

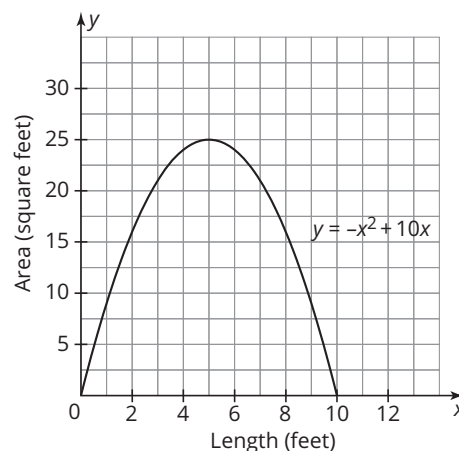
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Up and Down or Down and Up

The shape that a quadratic function forms when graphed is called a parabola. A **parabola** is a smooth curve in a U-shape that has symmetry. The parabola can open upward, decreasing to a minimum point before increasing, or can open downward, increasing to a maximum point before decreasing. The domain of a quadratic function is all real numbers. The range of a quadratic function is all real numbers greater than or equal to the minimum y -value or less than or equal to the maximum y -value. The graph of a quadratic function has one y -intercept and, at most, 2 x -intercepts.

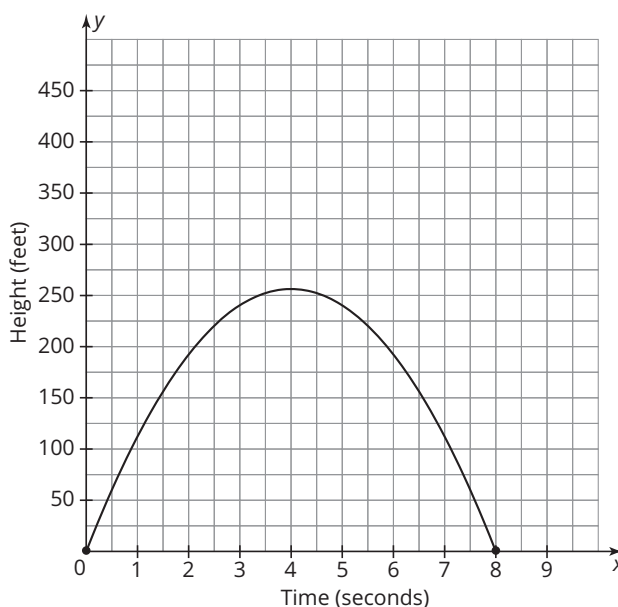
Quadratic functions model area because area is measured in square units.

For example, suppose you have 20 feet of fencing with which to enclose a rectangular area. The graph represents a quadratic function for the area of the rectangle given possible lengths of the rectangle. The maximum of the parabola is at the point (5, 25). It has x-intercepts at (0, 0) and (10, 0), and a y-intercept at (0, 0).



A **vertical motion model** is a quadratic equation that models the height of an object at a given time. The equation is of the form $y = -16t^2 + v_0t + h_0$, where y represents the height of the object in feet, t represents the time in seconds that the object has been moving, v_0 represents the initial vertical velocity of the object in feet per second, and h_0 represents the initial height of the object in feet.

For example, suppose a firework is launched into the air from the ground with a vertical velocity of 128 feet per second. The function that describes the height of the firework in terms of time is $g(t) = -16t^2 + 128t$.



The x-intercepts of a graph of a quadratic function are also called the zeros of the quadratic function. The x-intercepts, or zeros, of $g(t) = -16t^2 + 128t$ are (0, 0) and (8, 0).

When an equation is used to model a situation, the x-intercepts are referred to as **roots**. The roots of an equation indicate where the graph of the equation crosses the x-axis.

Endless Forms Most Beautiful

First differences are the differences between successive output values when successive input values have a difference of 1. **Second differences** are the differences between consecutive values of first differences. Linear functions have constant first differences and second differences of 0. Quadratic functions have changing first differences and constant second differences.

Linear function: $f(x) = -x + 2$

x	$f(x)$	First Differences	Second Differences
0	2		
1	1	-1	
2	0	-1	0
3	-1	-1	0
4	-2	-1	0

Quadratic function: $f(x) = 2x^2 - 3x$

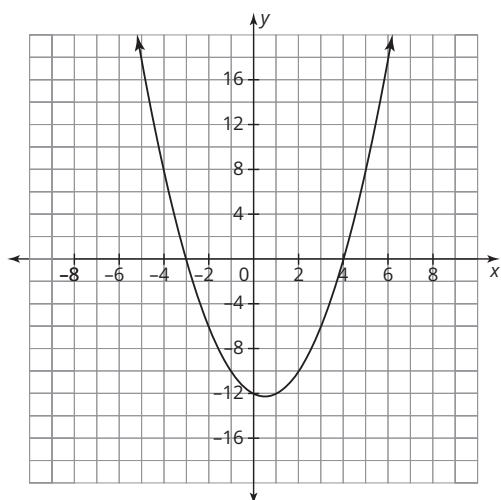
x	$f(x)$	First Differences	Second Differences
0	0		
1	-1	-1	
2	2	3	4
3	9	7	4
4	20	11	4

A quadratic function written in standard form, which is also called the **general form of a quadratic function**, is in the form, $f(x) = ax^2 + bx + c$, where $a \neq 0$. In this form, a and b represent numerical coefficients and c represents a constant. A quadratic function written in factored form is in the form $f(x) = a(x - r_1)(x - r_2)$, where $a \neq 0$ and r_1 and r_2 represent the roots.

When the leading coefficient a is negative, the graph of the quadratic function opens downward and has a maximum. When a is positive, the graph of the quadratic function opens upward and has a minimum. When a quadratic function is written in general form, the constant c is the y -intercept.

The **vertex** of a parabola is the lowest or highest point on the graph of the quadratic function. The **axis of symmetry** of a parabola is the vertical line that passes through the vertex and divides the parabola into two mirror images.

For a quadratic function in factored form, the equation for the axis of symmetry is given by $x = \frac{r_1 + r_2}{2}$. For a quadratic function in general form, the equation for the axis of symmetry is $x = \frac{-b}{2a}$.



The graph shown represents the function $f(x) = x^2 - x - 12$. The axis of symmetry is $x = -\frac{(-1)}{2(1)} = \frac{1}{2}$. The vertex is $(\frac{1}{2}, -12\frac{1}{4})$. The x -intercepts, or zeros, of the function are $x = -3$ and $x = 4$, so the function can be written in factored form as $f(x) = (x + 3)(x - 4)$. The y -intercept is $(0, -12)$. The domain of the function is all real numbers and the range is all real numbers greater than or equal to $-12\frac{1}{4}$. The graph has an interval of decrease from $-\infty$ to $\frac{1}{2}$ and an interval of increase from $\frac{1}{2}$ to ∞ .

You can use the fact that the graph of a quadratic function is symmetric along the axis of symmetry to determine a second point on the parabola given a point on the parabola, and to determine the axis of symmetry given two symmetric points on the parabola.

For example, the vertex of a parabola is $(3, 5)$. A point on the parabola is $(0, 3)$.
Another point on the parabola is $(6, 3)$.

$$\begin{aligned} \frac{0 + a}{2} &= 3 \\ 0 + a &= 6 \\ a &= 6 \end{aligned}$$

Suppose two symmetric points on a parabola are $(-7, 20)$ and $(4, 20)$. The axis of symmetry is $x = -\frac{3}{2}$ because $\frac{-7 + 4}{2} = -\frac{3}{2}$.

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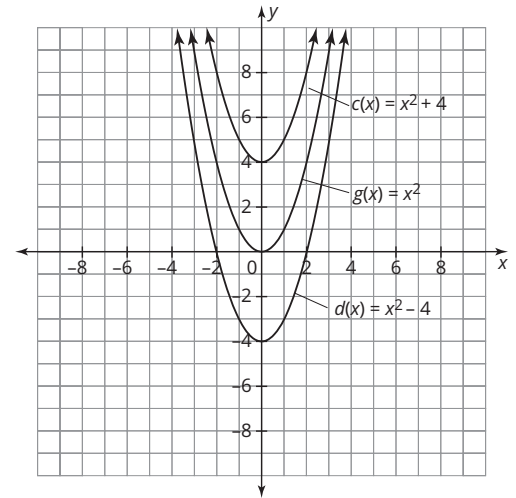
More Than Meets the Eye

You can use function transformation form, $g(x) = A \cdot f(B(x - C)) + D$, to transform quadratic functions.

Vertical translations are performed on a basic quadratic function $g(x) = x^2$ by adding a constant to or subtracting a constant from the function. Adding to the function translates it up, and subtracting translates it down. Horizontal translations are performed on the basic quadratic function $g(x) = x^2$ by adding a constant to or subtracting a constant from the argument, x , of the function. Adding to the argument translates the function to the left, and subtracting from the argument translates the function to the right.

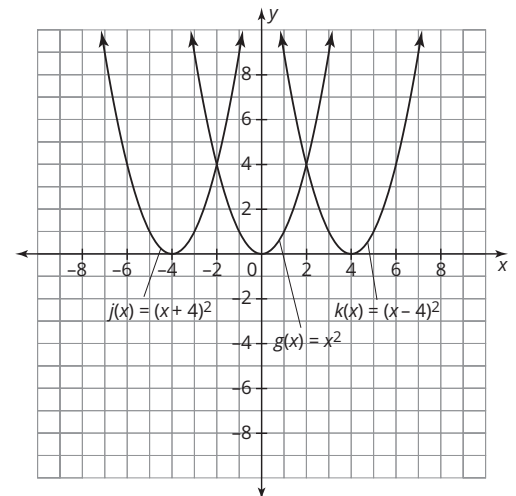
Vertical translations

- $g(x) = x^2$ basic function
 $c(x) = g(x) + 4$ $g(x)$ translated 4 units up, so
 $(x, y) \longrightarrow (x, y + 4)$.
 $d(x) = g(x) - 4$ $g(x)$ translated 4 units down, so
 $(x, y) \longrightarrow (x, y - 4)$.



Horizontal translations

- $g(x) = x^2$ basic function
 $j(x) = g(x + 4)$ $g(x)$ translated 4 units left,
so $(x, y) \longrightarrow (x - 4, y)$.
 $k(x) = g(x - 4)$ $g(x)$ translated 4 units right,
so $(x, y) \longrightarrow (x + 4, y)$.



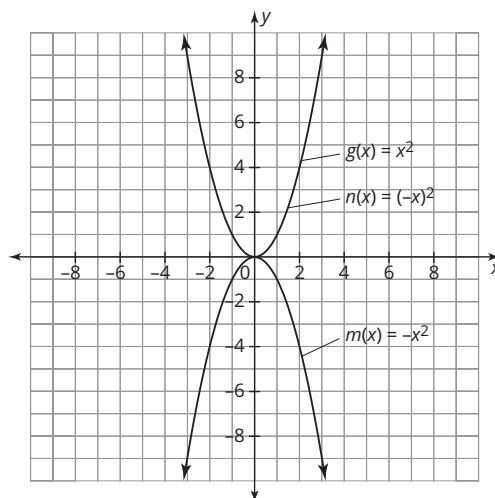
Multiplying the basic quadratic function by -1 results in a reflection across the line $y = 0$. Multiplying the argument of the basic quadratic function by -1 results in a reflection across the line $x = 0$, which ends up being the same as the original function because quadratic functions have a vertical axis of symmetry.

Reflections

$g(x) = x^2$ basic function

$m(x) = -g(x)$ $g(x)$ is reflected across $y = 0$,
so $(x, y) \longrightarrow (x, -y)$.

$n(x) = g(-x)$ $g(x)$ is reflected across $x = 0$,
so $(x, y) \longrightarrow (-x, y)$.



A vertical dilation of a function is a transformation in which the y -coordinate of every point on the graph of the function is multiplied by a common factor called the dilation factor. A vertical dilation stretches or shrinks the graph of a function vertically. For the transformed basic quadratic function, $g(x) = A \cdot f(x)$, when $|A| > 1$, the graph of $f(x)$ is stretched vertically. When $0 < |A| < 1$, $f(x)$ shrinks vertically. You can use the coordinate notation shown to indicate a vertical dilation.

$(x, y) \longrightarrow (x, Ay)$, where A is the dilation factor.

A horizontal dilation of a function is a transformation in which the x -coordinate of every point on the graph of the function is multiplied by a dilation factor. For the transformed basic quadratic function, $g(x) = f(Bx)$, when $|B| > 1$, the graph of the function is compressed horizontally. When $0 < |B| < 1$, the function is stretched horizontally. You can use the coordinate notation shown to indicate a horizontal dilation.

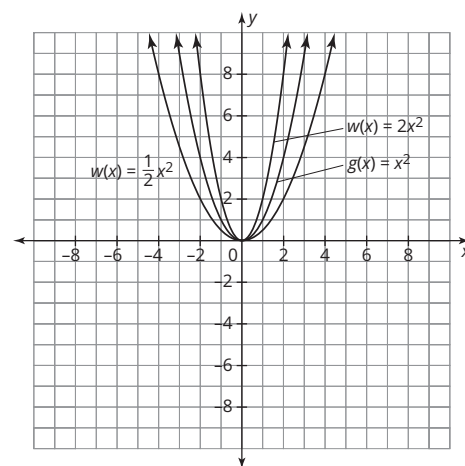
$(x, y) \longrightarrow \left(\frac{1}{|B|}x, y\right)$, where $\frac{1}{|B|}$ is the dilation factor.

Vertical dilations

$g(x) = x^2$ basic function

$v(x) = 2g(x)$ $g(x)$ stretched by a dilation factor of 2,
so $(x, y) \rightarrow (x, 2y)$.

$w(x) = \frac{1}{2}g(x)$ $g(x)$ shrunk by a dilation factor of $\frac{1}{2}$,
so $(x, y) \rightarrow (x, \frac{1}{2}y)$.

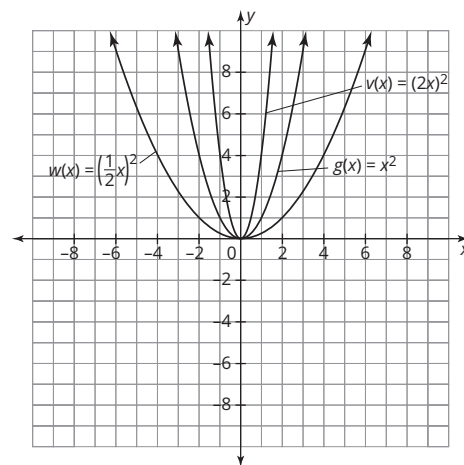


Horizontal dilations

$g(x) = x^2$ basic function

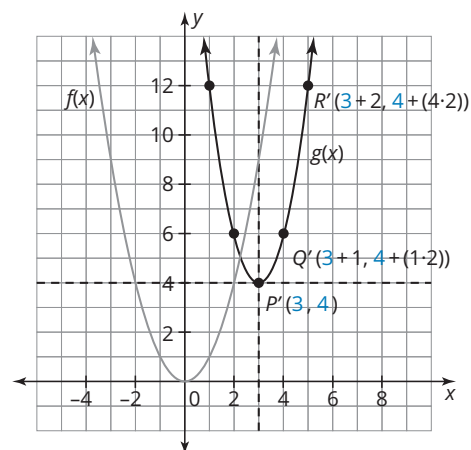
$v(x) = g(2x)$ $g(x)$ compressed by a dilation factor of $\frac{1}{2}$,
so $(x, y) \rightarrow (\frac{1}{2}x, y)$.

$w(x) = g(\frac{1}{2}x)$ $g(x)$ stretched by a dilation factor of 2,
so $(x, y) \rightarrow (2x, y)$.



Given $y = f(x)$ is the basic function, you can use reference points to graph $y = A \cdot f(B(x - C)) + D$ without the use of technology. Any point (x, y) on $f(x)$ maps to the point $(\frac{1}{B}x + C, Ay + D)$.

Given $f(x) = x^2$, the function $g(x) = 2f(x - 3) + 4$ has been graphed.



A quadratic function written in **vertex form** is in the form $f(x) = a(x - h)^2 + k$, where $a \neq 0$. The variable h represents the x -coordinate of the vertex. The variable k represents the y -coordinate of the vertex.

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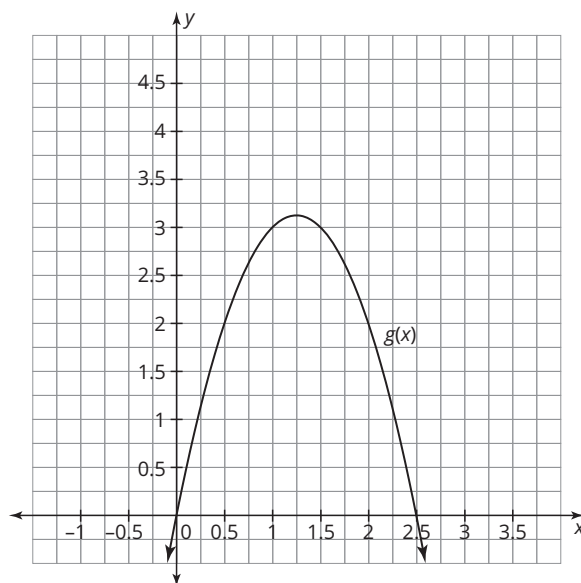
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You Lose Some, You Lose Some

You can use different representations of quadratic functions to analyze their key characteristics.

For example, the functions $f(x)$ shown with a table and $g(x)$ shown with a graph can be compared to determine which quadratic function has the greater maximum.

x	$f(x)$
0	0
0.5	1
0.75	1.125
1	1
1.5	0



From the table, you can determine the maximum of $f(x)$ is $(0.75, 1.125)$. From the graph, you can determine that the maximum of $g(x)$ is greater than $y = 3$. Therefore, $g(x)$ has a greater maximum than $f(x)$.