Skills Practice

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I. Modeling with Quadratic Functions

A. Write a quadratic function in standard form that represents each area as a function of the width. Remember to define your variables.

1. A builder is designing a rectangular parking lot. She has 300 feet of fencing to enclose the parking lot around three sides.

2. Aiko is enclosing a new rectangular flower garden with a rabbit garden fence. She has 40 feet of fencing.

3. Pedro is building a rectangular sandbox for the community park. The materials available limit the perimeter of the sandbox to at most 100 feet.

4. Lea is designing a rectangular quilt. She has 16 feet of piping to finish the quilt around three sides.

5. Kiana is making a rectangular vegetable garden alongside her home. She has 24 feet of fencing to enclose the garden around the three open sides.

6. Nelson is building a rectangular ice rink for the community park. The materials available limit the perimeter of the ice rink to at most 250 feet.

B. Use technology to determine the absolute maximum of each function. Describe what the *x*- and *y*-coordinates of this point represent in terms of the problem situation.

1. A builder is designing a rectangular parking lot. He has 400 feet of fencing to enclose the parking lot around three sides. Let x = the width of the parking lot. Let A = the area of the parking lot. The function $A(x) = -2x^2 + 400x$ represents the area of the parking lot as a function of the width.

2. Joelle is enclosing a portion of her yard to make a pen for her ferrets. She has 20 feet of fencing. Let x = the width of the pen. Let A = the area of the pen. The function $A(x) = -x^2 + 10x$ represents the area of the pen as a function of the width.

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3. A baseball is thrown upward from a height of 5 feet with an initial velocity of 42 feet per second. Let t = the time in seconds after the baseball is thrown. Let h = the height of the baseball. The quadratic function $h(t) = -16t^2 + 42t + 5$ represents the height of the baseball as a function of time.

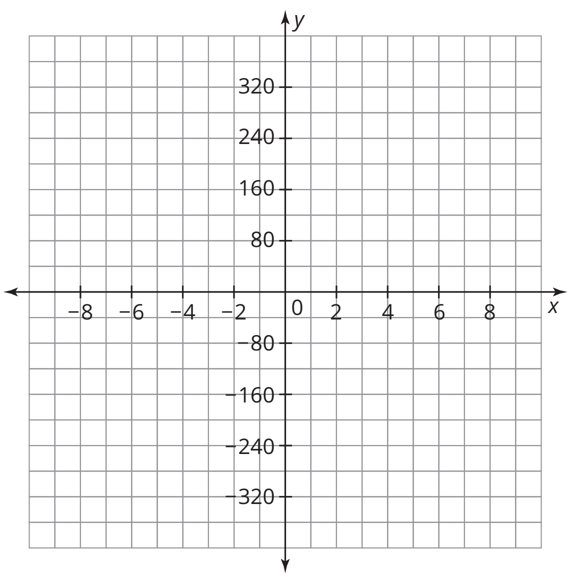
4. Hector is standing on top of a playground set at a park. He throws a water balloon upward from a height of 12 feet with an initial velocity of 25 feet per second. Let t = the time in seconds after the balloon is thrown. Let h = the height of the balloon. The quadratic function $h(t) = -16t^2 + 25t + 12$ represents the height of the balloon as a function of time.

5. Franco is building a rectangular roller-skating rink at the community park. The materials available limit the perimeter of the skating rink to at most 180 feet. Let x = the width of the skating rink. Let A = the area of the skating rink. The function $A(x) = -x^2 + 90x$ represents the area of the skating rink as a function of the width.

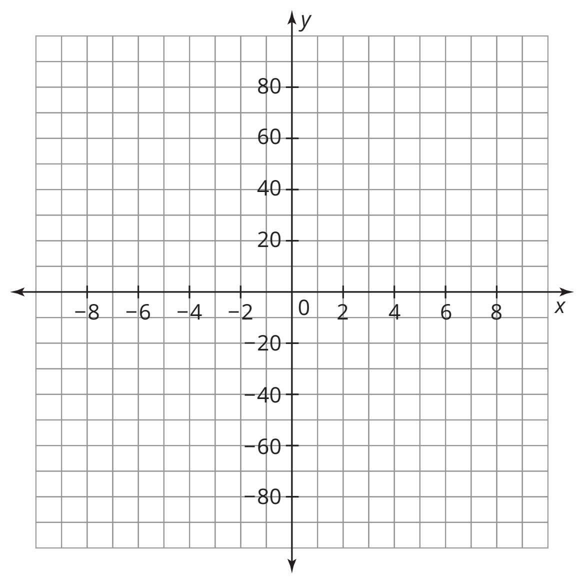
6. A football is thrown upward from a height of 6 feet with an initial velocity of 65 feet per second. Let t = the time in seconds after the football is thrown. Let h = the height of the football. The quadratic function $h(t) = -16t^2 + 65t + 6$ represents the height of the football as a function of time.

C. Graph the function that represents each problem situation. Identify the absolute maximum, zeros, and the domain and range of the function in terms of both the graph and problem situation. Round your answers to the nearest hundredth, if necessary.

1. A model rocket is launched from the ground with an initial velocity of 120 feet per second. The function $g(t) = -16t^2 + 120t$ represents the height of the rocket, $g(t)$, t seconds after it was launched.



2. A model rocket is launched from the ground with an initial velocity of 60 feet per second. The function $g(t) = -16t^2 + 60t$ represents the height of the rocket, $g(t)$, t seconds after it was launched.



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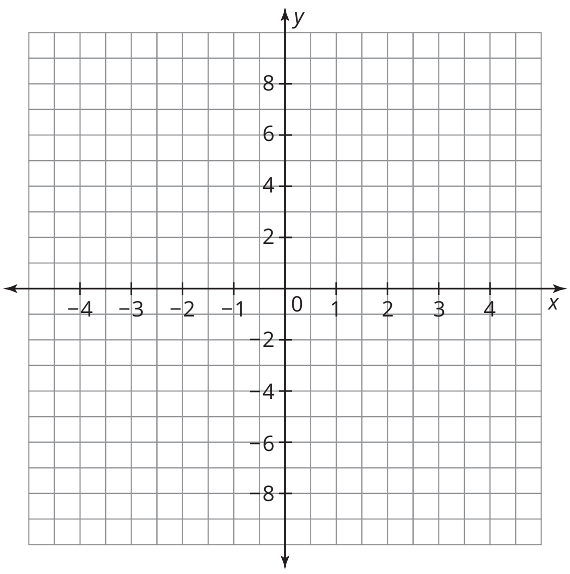
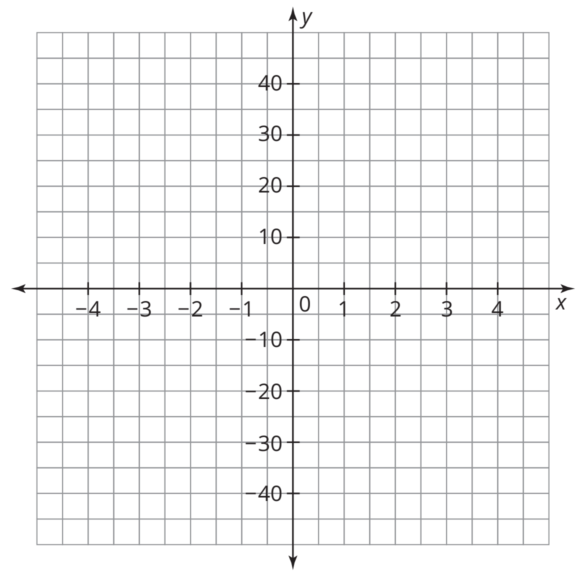
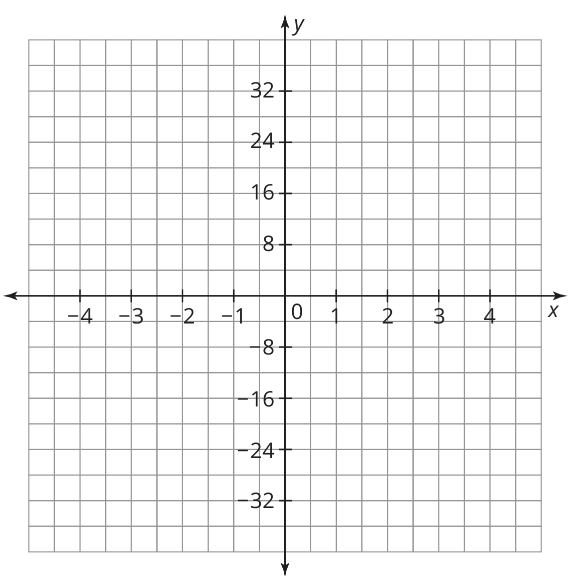
3. A baseball is thrown in the air from a height of 5 feet with an initial vertical velocity of 15 feet per second. The function $g(t) = -16t^2 + 15t + 5$ represents the height of the baseball, $g(t)$, t seconds after it was launched.



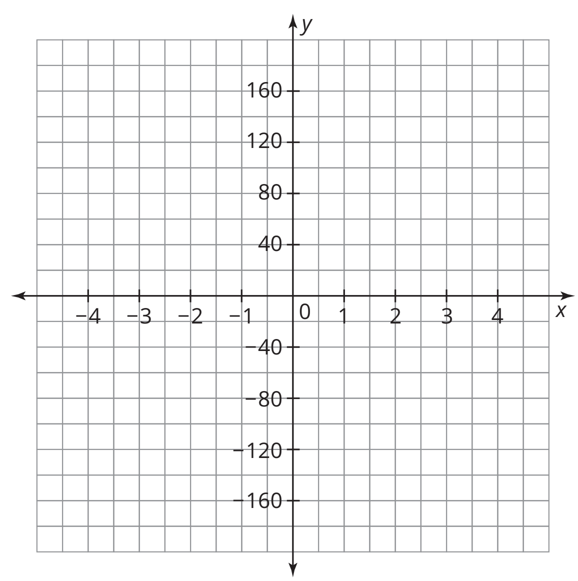
4. A football is thrown in the air from a height of 6 feet with an initial vertical velocity of 50 feet per second. The function $g(t) = -16t^2 + 50t + 6$ represents the height of the football, $g(t)$, t seconds after it was launched.



5. A tennis ball is dropped from a height of 25 feet. The initial velocity of an object that is dropped is 0 feet per second. The function $g(t) = -16t^2 + 25$ represents the height of the tennis ball, $g(t)$, t seconds after it was dropped.



6. A tennis ball is dropped from a height of 150 feet. The initial velocity of an object that is dropped is 0 feet per second. The function $g(t) = -16t^2 + 150$ represents the height of the tennis ball, $g(t)$, t seconds after it was dropped.



D. Write a function that represents the vertical motion described in each problem situation.

1. A catapult hurls a watermelon from a height of 36 feet at an initial velocity of 82 feet per second.

2. A catapult hurls a cantaloupe from a height of 12 feet at an initial velocity of 47 feet per second.

3. A catapult hurls a pineapple from a height of 49 feet at an initial velocity of 110 feet per second.

4. A basketball is thrown from a height of 7 feet at an initial velocity of 58 feet per second.

5. A soccer ball is thrown from a height of 25 feet at an initial velocity of 46 feet per second.

6. A football is thrown from a height of 6 feet at an initial velocity of 74 feet per second.

E. Identify the vertex and the equation of the axis of symmetry for each vertical motion model.

1. A catapult hurls a grapefruit from a height of 24 feet at an initial velocity of 80 feet per second. The function $h(t) = -16t^2 + 80t + 24$ represents the height of the grapefruit $h(t)$ in terms of time t.

2. A catapult hurls a pumpkin from a height of 32 feet at an initial velocity of 96 feet per second. The function $h(t) = -16t^2 + 96t + 32$ represents the height of the pumpkin $h(t)$ in terms of time t.

3. A catapult hurls a watermelon from a height of 40 feet at an initial velocity of 64 feet per second. The function $h(t) = -16t^2 + 64t + 40$ represents the height of the watermelon $h(t)$ in terms of time t.

4. A baseball is thrown from a height of 6 feet at an initial velocity of 32 feet per second. The function $h(t) = -16t^2 + 32t + 6$ represents the height of the baseball $h(t)$ in terms of time t.

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5. A softball is thrown from a height of 20 feet at an initial velocity of 48 feet per second. The function $h(t) = -16t^2 + 48t + 20$ represents the height of the softball $h(t)$ in terms of time t.

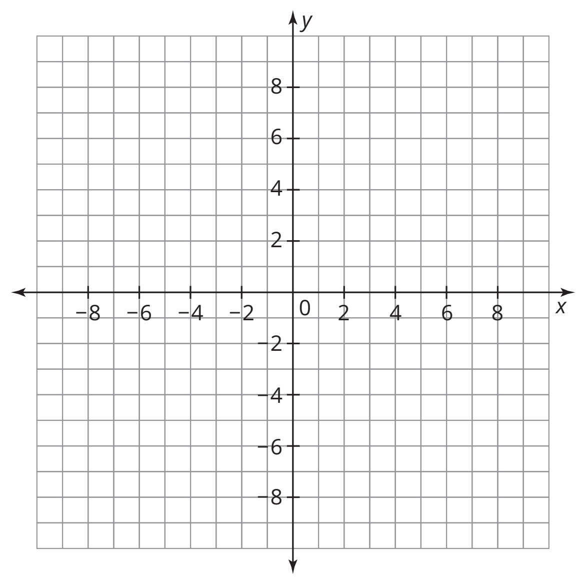
6. A rocket is launched from the ground at an initial velocity of 112 feet per second. The function $h(t) = -16t^2 + 112t$ represents the height of the rocket $h(t)$ in terms of time t.

II. Comparing Linear, Quadratic, and Exponential Functions

A. Graph each table of values. Describe the type of function represented by the graph.

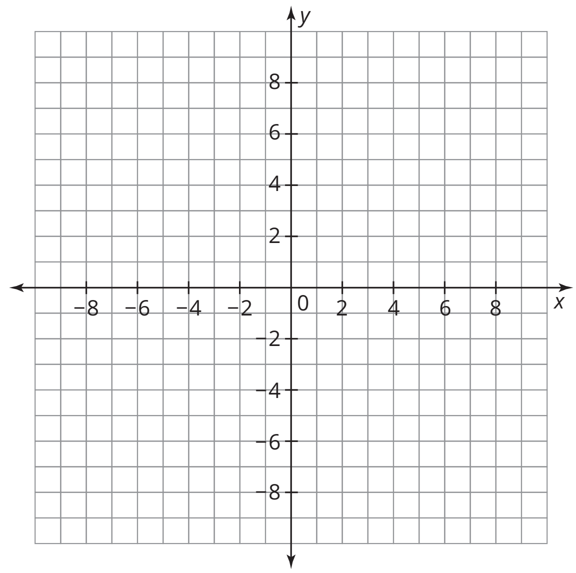
1.

x	*y*
−4	7
−2	6
0	5
2	4
4	3



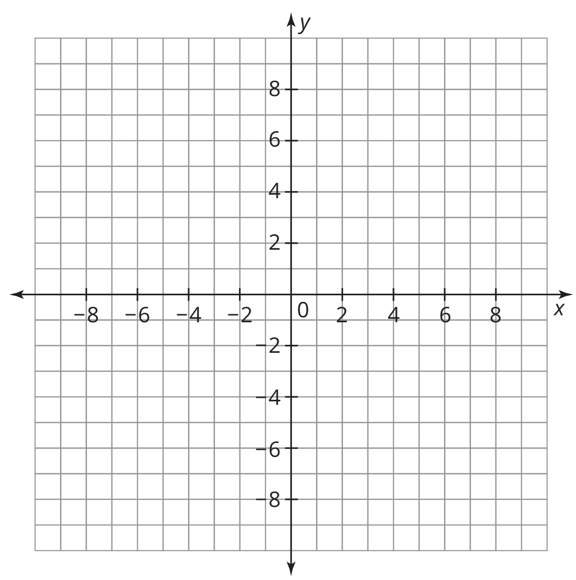
2.

x	y
−3	$\frac{1}{8}$
−2	$\frac{1}{4}$
−1	$\frac{1}{2}$
0	1
1	2



3.

x	y
−2	−8
0	0
2	4
4	4
6	0



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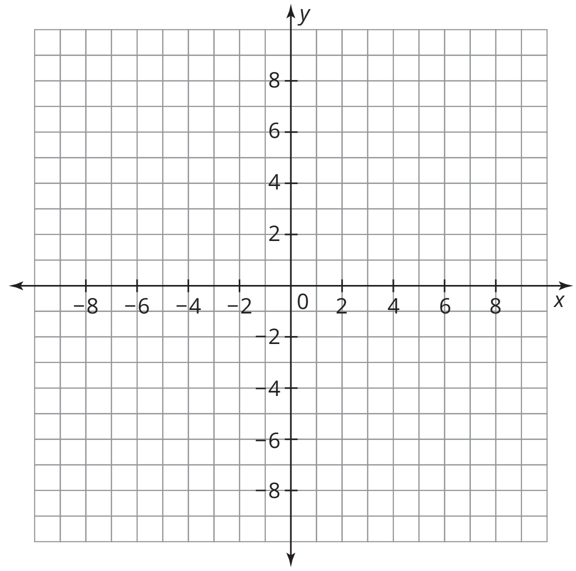
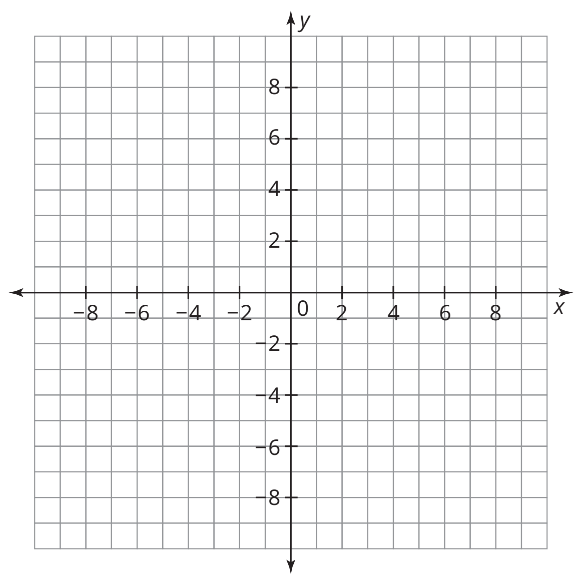
4.

x	y
−2	$\frac{1}{9}$
−1	$\frac{1}{3}$
0	1
1	3
2	9



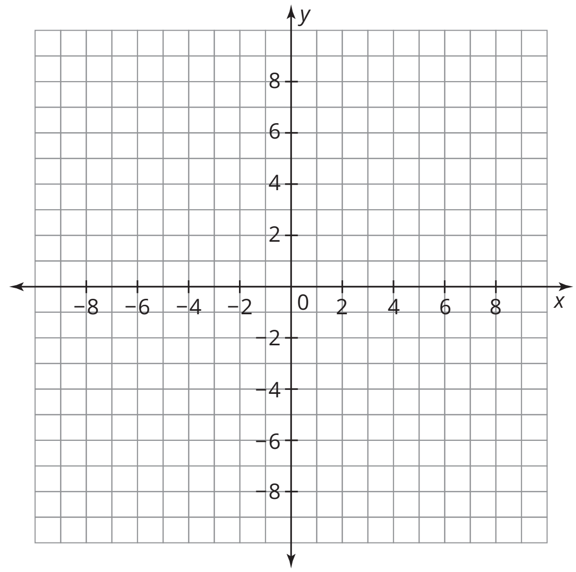
5.

x	y
1	6
2	3
3	0
4	−3
5	−6



6.

x	*y*
−3	−9
0	0
3	3
6	0
9	−9



B. Calculate the first and second differences for each table of values. Describe the type of function represented by the table.

1.

x	*y*	First Differences	Second Differences
−2	−6		
−1	−3		
0	0		
1	3		
2	6		

2.

x	*y*	First Differences	Second Differences
−2	12		
−1	3		
0	0		
1	3		
2	12		

3.

x	*y*	First Differences	Second Differences
−3	3		
−2	4		
−1	5		
0	6		
1	7		

4.

x	*y*	First Differences	Second Differences
−1	1		
0	0		
1	3		
2	10		
3	21		

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5.

x	*y*	First Differences	Second Differences
−4	−48		
−3	−27		
−2	−12		
−1	−3		
0	0		

6.

x	*y*	First Differences	Second Differences
−1	10		
0	8		
1	6		
2	4		
3	2		

C. Calculate the average rate of change of the functions $f(x) = x$, $g(x) = x^2$, and $h(x) = 2^x$ for each interval.

1. [−1, 0]

2. [−2, 2]

3. [0, 3]

4. [2, 4]

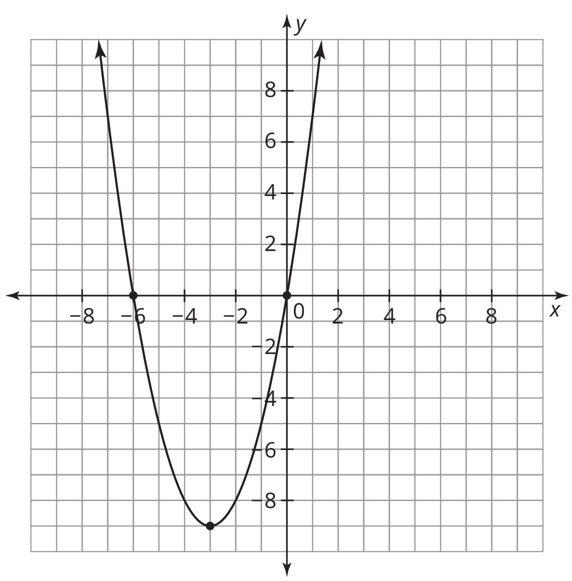
5. [0, 5]

6. [4, 5]

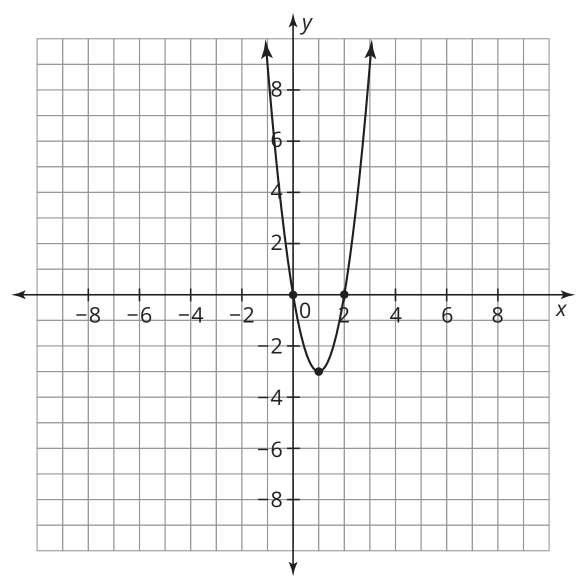
III. Identifying Characteristics of Quadratic Functions

A. Identify the intervals of increase and decrease for each function.

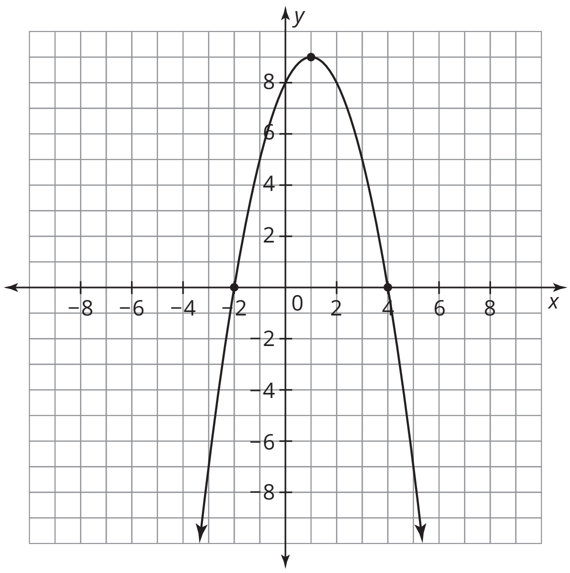
1. $f(x) = x^2 + 6x$



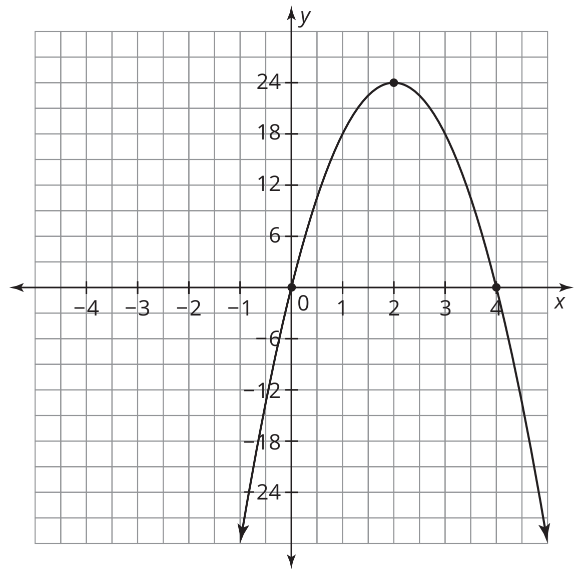
2. $f(x) = 3x^2 - 6x$



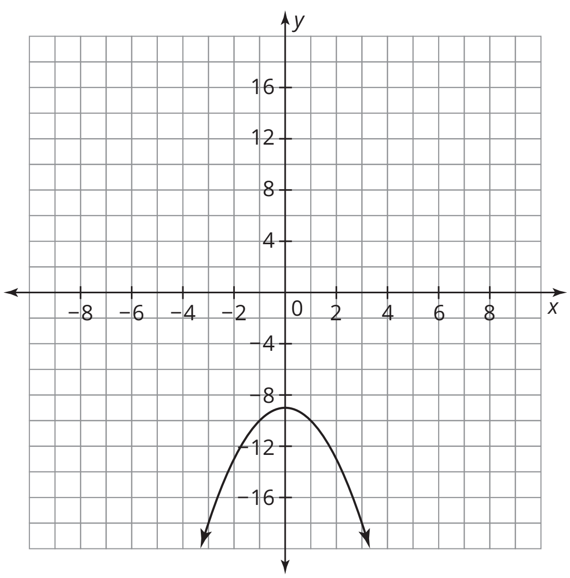
3. $f(x) = -x^2 + 2x + 8$



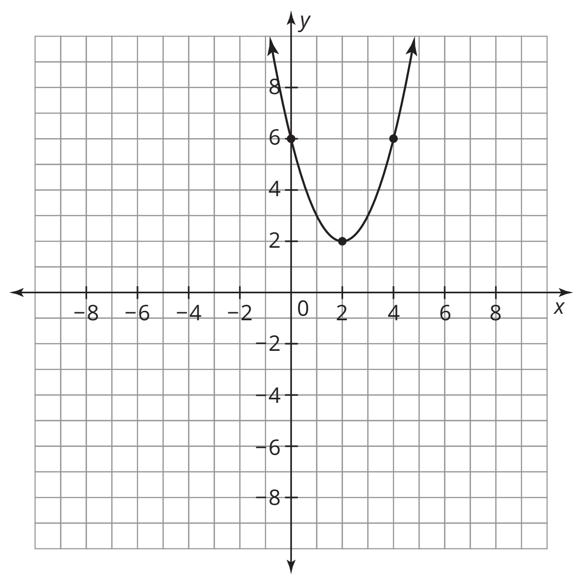
4. $f(x) = -6x^2 + 24x$



5. $f(x) = -x^2 - 9$



6. $f(x) = x^2 - 4x + 6$



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B. Determine the x-intercepts of each quadratic function in factored form.

1. $f(x) = (x - 2)(x - 8)$

2. $f(x) = (x + 1)(x - 6)$

3. $f(x) = 3(x + 4)(x - 2)$

4. $f(x) = 0.25(x - 1)(x - 12)$

5. $f(x) = 0.5(x + 15)(x + 5)$

6. $f(x) = 4(x - 1)(x - 9)$

C. Determine the axis of symmetry of each parabola.

1. The x-intercepts of a parabola are (3, 0) and (9, 0).

2. The x-intercepts of a parabola are (−3, 0) and (1, 0).

3. The x-intercepts of a parabola are (−12, 0) and (−2, 0).

4. Two symmetric points on a parabola are (−1, 4) and (5, 4).

5. Two symmetric points on a parabola are (−4, 8) and (2, 8).

6. Two symmetric points on a parabola are (3, 1) and (15, 1).

D. Determine the vertex of each parabola.

1. $f(x) = x^2 + 2x - 15$
axis of symmetry: $x = -1$

2. $f(x) = x^2 - 8x + 7$
axis of symmetry: $x = 4$

3. $f(x) = x^2 + 4x - 12$
x-intercepts: (2, 0) and (−6, 0)

4. $f(x) = -x^2 - 14x - 45$
x-intercepts: (−9, 0) and (−5, 0)

5. $f(x) = -x^2 + 8x + 20$
two symmetric points on the parabola:
(−1, 11) and (9, 11)

6. $f(x) = -x^2 + 16$
two symmetric points on the parabola:
(−3, 7) and (3, 7)

E. Determine another point on each parabola.

1. The axis of symmetry is $x = 3$. A point on the parabola is (1, 4).

2. The axis of symmetry is $x = -4$. A point on the parabola is (0, 6).

3. The axis of symmetry is $x = 1$. A point on the parabola is (−3, 2).

4. The vertex is (5, 2). A point on the parabola is (3, −1).

5. The vertex is (−1, 6). A point on the parabola is (2, 3).

6. The vertex is (3, −1). A point on the parabola is (4, 1).

F. Determine the vertex of each quadratic function given in vertex form.

1. $f(x) = (x - 3)^2 + 8$

2. $f(x) = (x + 4)^2 + 2$

3. $f(x) = -2(x - 1)^2 - 8$

4. $f(x) = \frac{1}{2}(x - 2)^2 + 6$

5. $f(x) = -(x + 9)^2 - 1$

6. $f(x) = (x - 5)^2$

G. Identify the form of each quadratic function as either standard form, factored form, or vertex form. Then state all you know about the quadratic function's key characteristics, based only on the given equation of the function.

1. $f(x) = 5(x - 3)^2 + 12$

2. $f(x) = -(x - 8)(x - 4)$

3. $f(x) = -3x^2 + 5x$

4. $f(x) = \frac{2}{3}(x + 6)(x - 1)$

5. $f(x) = -(x + 2)^2 - 7$

6. $f(x) = 2x^2 - 1$

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IV. Writing Quadratic Functions

A. Write a quadratic function in factored form with each set of given characteristics.

1. Write a quadratic function that represents a parabola that opens downward and has x-intercepts $(-2, 0)$ and $(5, 0)$.

2. Write a quadratic function that represents a parabola that opens downward and has x-intercepts $(2, 0)$ and $(14, 0)$.

3. Write a quadratic function that represents a parabola that opens upward and has x-intercepts $(-8, 0)$ and $(-1, 0)$.

4. Write a quadratic function that represents a parabola that opens upward and has x-intercepts $(3, 0)$ and $(7, 0)$.

5. Write a quadratic function that represents a parabola that opens downward and has x-intercepts $(-5, 0)$ and $(2, 0)$.

6. Write a quadratic function that represents a parabola that opens upward and has x-intercepts $(-12, 0)$ and $(-4, 0)$.

B. Determine the x-intercepts for each function using technology. Write the function in factored form.

1. $f(x) = x^2 - 8x + 7$

2. $f(x) = 2x^2 - 10x - 48$

3. $f(x) = -x^2 - 20x - 75$

4. $f(x) = x^2 + 8x + 12$

5. $f(x) = -3x^2 - 9x + 12$

6. $f(x) = x^2 - 6x$

C. Use technology to determine the vertex of each quadratic function given in standard form. Rewrite the function in vertex form.

1. $f(x) = x^2 - 6x - 27$

2. $f(x) = -x^2 - 2x + 15$

3. $f(x) = 2x^2 - 4x - 6$

4. $f(x) = x^2 - 10x + 24$

5. $f(x) = -x^2 + 15x - 54$

6. $f(x) = -2x^2 - 14x - 12$

D. Write an equation for a quadratic function that satisfies each set of given characteristics.

1. The vertex is (–1, 4) and the parabola opens down.

2. The x-intercepts are −3 and 4 and the parabola opens down.

2. The vertex is (3, −2) and the parabola opens up.

4. The vertex is (0, 8) and the parabola opens up.

5. The x-intercepts are 5 and 12 and the parabola opens up.

6. The x-intercepts are 0 and 7 and the parabola opens down.

7. The vertex is (–2, −3) and the parabola passes through the point (1, 6).

8. The vertex is (0, 0) and the parabola passes through the point (–2, −8).

9. The function has zeros (6, 0) and (−4, 0), and the parabola passes through the point (0, 8).

10. The vertex is (−4, 0) and the parabola passes through the point (−6, 12).

11. The vertex is (0, −8) and the parabola passes through the point (4, 0).

12. The function has zeros (5, 0) and (–1, 0), and the parabola passes through the point (1, −8).

V. Transforming Quadratic Functions

A. Describe the transformation performed on each function $g(x)$ to result in $d(x)$.

1. $g(x) = x^2$
$d(x) = x^2 - 5$

2. $g(x) = x^2$
$d(x) = -x^2$

3. $g(x) = x^2$
$d(x) = x^2 + 2$

4. $g(x) = x^2$
$d(x) = (x + 4)^2$

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5. $g(x) = 3x^2$
$d(x) = 3x^2 + 6$

6. $g(x) = x^2$
$d(x) = (-x)^2$

7. $g(x) = \frac{1}{2}x^2$
$d(x) = \frac{1}{2}x^2 - 1$

8. $g(x) = x^2$
$d(x) = (x - 8)^2$

9. $g(x) = (x + 2)^2$
$d(x) = (x + 2)^2 - 3$

10. $g(x) = x^2 + 2$
$d(x) = -(x^2 + 2)$

11. $g(x) = x^2$
$d(x) = (x + 1)^2$

12. $g(x) = x^2 - 5$
$d(x) = (-x)^2 - 5$

13. $g(x) = x^2 - 7$
$d(x) = (x + 2)^2 - 7$

14. $g(x) = -(x - 2)^2$
$d(x) = -(x - 2)^2 + 5$

15. $g(x) = x^2 + 8$
$d(x) = (x + 3)^2 + 8$

16. $g(x) = \frac{2}{3}x^2 + 4$
$d(x) = \frac{2}{3}(-x)^2 + 4$

17. $g(x) = x^2 - 6$
$d(x) = (x - 5)^2 - 6$

18. $g(x) = 5x^2 - 7$
$d(x) = -(5x^2 - 7)$

B. Represent each function *n(x)* as a vertical dilation of *g(x)* using coordinate notation.

1. $g(x) = x^2$
$n(x) = 4x^2$

2. $g(x) = x^2$
$n(x) = \frac{1}{2}x^2$

3. $g(x) = -x^2$
$n(x) = -5x^2$

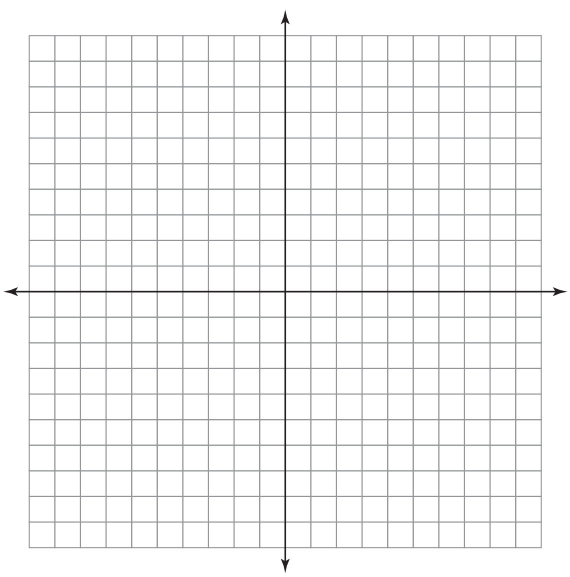
4. $g(x) = -x^2$
$n(x) = -\frac{3}{4}x^2$

5. $g(x) = (x + 1)^2$
$n(x) = 2(x + 1)^2$

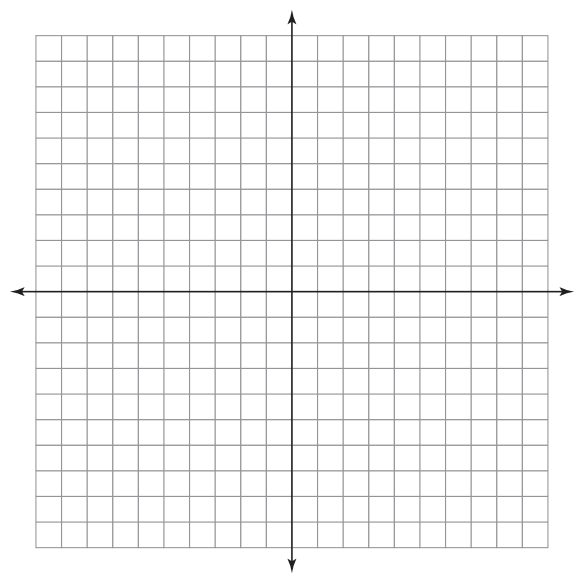
6. $g(x) = (x - 3)^2$
$n(x) = \frac{1}{2}(x - 3)^2$

C. Write an equation in vertex form for a function $g(x)$ with the given characteristics. Sketch a graph of each function $g(x)$.

1. The function $g(x)$ is quadratic.
The function $g(x)$ is continuous.
The graph of $g(x)$ is a horizontal reflection of the graph of $f(x) = x^2$.
The function $g(x)$ is translated 3 units up from $f(x) = -x^2$.



2. The function $g(x)$ is quadratic.
The function $g(x)$ is continuous.
The graph of $g(x)$ is a horizontal reflection of the graph of $f(x) = x^2$.
The function $g(x)$ is translated 2 units down and 5 units left from $f(x) = -x^2$.



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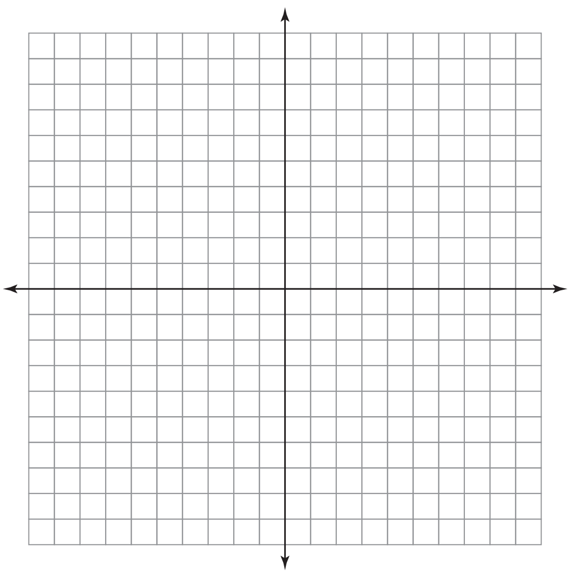
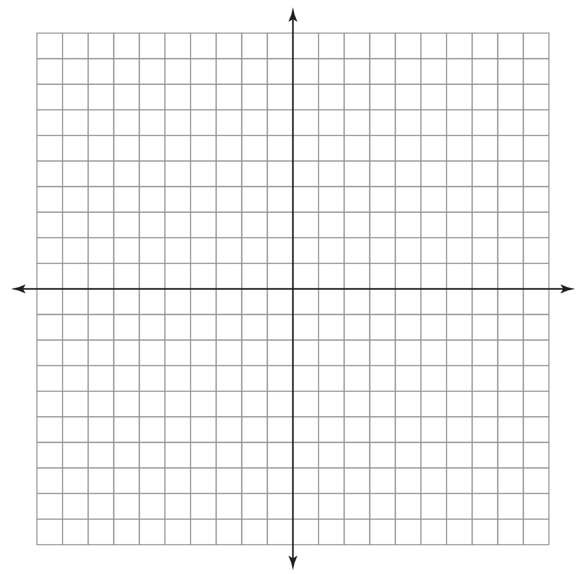
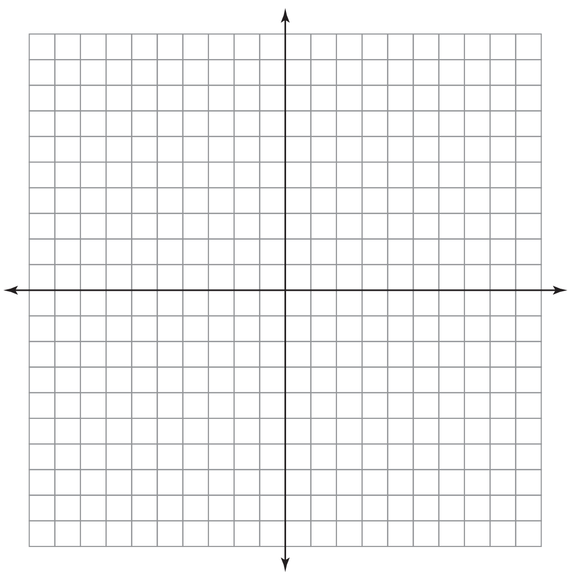
3. The function $g(x)$ is quadratic.
The function $g(x)$ is continuous.
The function $g(x)$ is vertically dilated with a dilation factor of 6.
The function $g(x)$ is translated 1 unit up and 4 units right from $f(x) = 6x^2$.



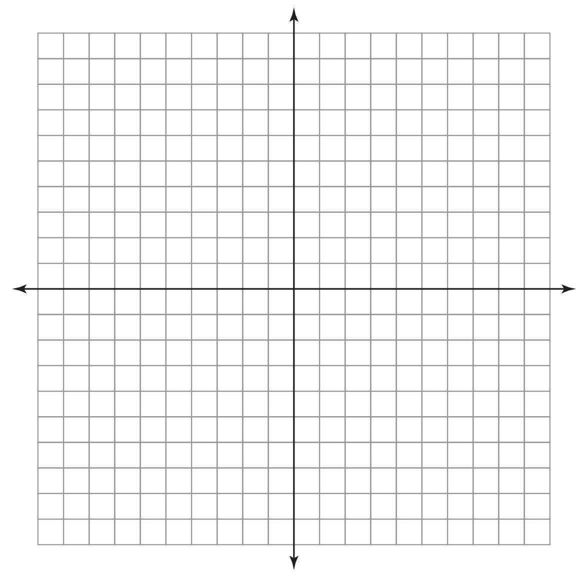
4. The function $g(x)$ is quadratic.
The function $g(x)$ is continuous.
The function $g(x)$ is vertically dilated with a dilation factor of $\frac{1}{2}$.
The function $g(x)$ is translated 2 units down and 6 units left from $f(x) = \frac{1}{2}x^2$.



5. The function $g(x)$ is quadratic.
The function $g(x)$ is continuous.
The graph of $g(x)$ is a horizontal reflection of the graph of $f(x) = x^2$.
The function $g(x)$ is vertically dilated with a dilation factor of 3.
The function $g(x)$ is translated 2 units down and 4 units right from $f(x) = -3x^2$.



6. The function $g(x)$ is quadratic.
The function $g(x)$ is continuous.
The function $g(x)$ is vertically dilated with a dilation factor of $\frac{1}{4}$.
The function $g(x)$ is translated 3 units up and 2 units left from $f(x) = \frac{1}{4}x^2$.



D. Describe the transformation(s) necessary to translate the graph of the function $f(x) = x^2$ into the graph of each function $g(x)$.

1. $g(x) = x^2 + 7$

2. $g(x) = -x^2 - 4$

3. $g(x) = (x - 2)^2 + 8$

4. $g(x) = 4x^2 + 1$

5. $g(x) = \frac{2}{3}(x + 4)^2 - 9$

6. $g(x) = -(x - 6)^2 + 3$