Module 3 Topic 3 (M3T3) Introduction to Quadratic Functions

1. Building a perimeter:

Write a quadratic function in standard form $f(x) = ax^2 + bx + c$, that represents the area as a function of the width. $A(x) = ax^2 + bx + c$ where x = the width

Remember: Perimeter of a rectangle
$P = 2l + 2w$
l = length,
w = width
Area of a rectangle: $A = l \cdot w$

Review: Midpoint Formula
For two points (x_1, y_1) and (x_2, y_2)
M is the midpoint.

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

2. Enclosing 4 sides

Let x = the width
$P = 2l + 2w$

So $l = \frac{P - 2w}{2}$

$A(x) = l \cdot w$

Example: John is building a fence around a flower garden he is making. He has 200 feet of fence to enclose the garden. Write a quadratic function in standard form that represents the area as a function of the width.
Let x = the width
Since $P = 2l + 2w$, then $200 = 2l + 2x$
so the length is

$$l = \frac{200 - 2x}{2} = 100 - x$$

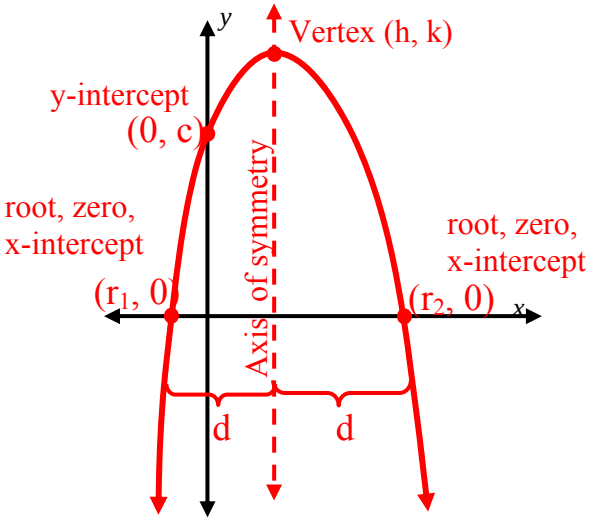
Area $A(x) = l \cdot w$ so $A(x) = (100-x)x$
So $A(x) = 100x - x^2$
Then $A(x) = -x^2 + 100x$

3. Enclosing 3 sides

Let x = the width
$P = l + 2w$
So $l = P - 2w$
$A(x) = l \cdot w$

Example: Jeff is building a dog run next to his house. He has 400 feet of fence to build the enclosure. Write a quadratic function in standard form that represents the area as a function of the width.
Let x = the width
Since $P = l + 2w$, then $400 = l + 2x$
so the length is $l = 400 - 2x$
Area $A(x) = l \cdot w$ so $A(x) = (400-2x)x$
So $A(x) = 400x - 2x^2$
Then $A(x) = -2x^2 + 400x$

4. Parts of the graph of a quadratic function



5. Vocabulary:

Absolute Maximum – Highest or lowest value of the function.
Axis of symmetry – line that divides the figure into identical halves.
Domain – input or x values
Range – output or y values
Vertex – the maximum (high point) or minimum (low point) of the graph
x-intercept – the point where the graph crosses the x axis.
y-intercept – the point where the graph crosses the y axis.
Zeros – x values that make the function/equation equal zero.

6. Interval notation – span of numbers.

Closed interval
inclusive (≤ or ≥)
[_____, _____]
low # high #

Open interval
not inclusive (< or >)
(_____, _____)
low # high #

Half open or half closed is allowed

7. Vertical Motion Model

$$h(t) = -16t^2 + v_0 t + h_0$$

v_0 = initial velocity or starting speed
h_0 = initial height or starting height

8. Standard Form

$f(x) = ax^2 + bx + c$

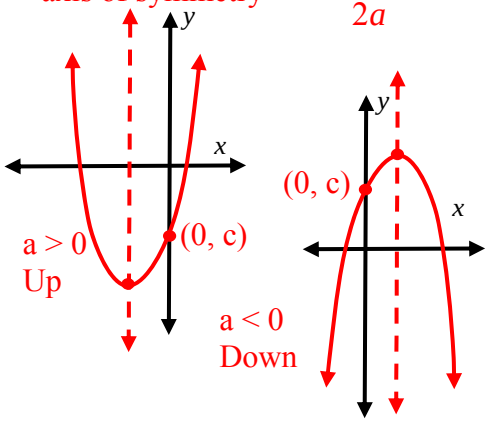
a – direction

a > 0 opens up

a < 0 opens down

c – y intercept (0, c)

axis of symmetry $x = -\frac{b}{2a}$



9. Vertex Form

$f(x) = a(x-h)^2 + k$

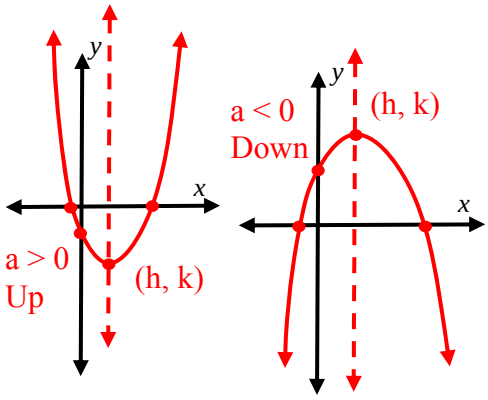
a – direction

a > 0 opens up

a < 0 opens down

Vertex – (h, k)

Axis of symmetry: x = h



10. Factored Form

$f(x) = a(x-r_1)(x-r_2)$

a – direction

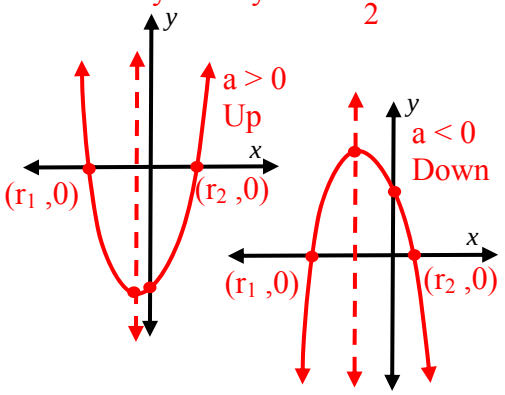
a > 0 opens up

a < 0 opens down

r_1 and r_2 roots, zeros or x-intercepts

(r_1, 0), (r_2 ,0)

Axis of symmetry: $x = \frac{r_1 + r_2}{2}$



12. Linear functions

f(x) = mx + b

1st difference - same

2nd difference - zero

Equation – linear

Graph – line

13. Quadratic functions

$f(x) = ax^2 + bx + c$

1st difference - varies

2nd difference - same

Equation – quadratic

Graph – parabola (∪ or ∩ shape)

14. Exponential functions

$f(x) = ab^x + q$

1st difference - varies

2nd difference – varies

The outputs have a common ratio.

Equation – exponential

Graph – a rounded corner

15. Average rate of change:

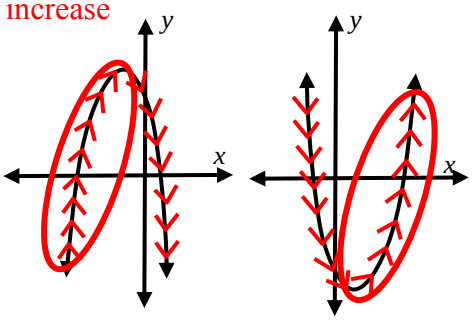
$\frac{f(b) - f(a)}{b - a}$

16. Calculating "a"

1. choose quadratic form
2. substitute the key point(s)
3. solve for "a"
4. write the equation by substituting the key point and the "a" value into the quadratic form.

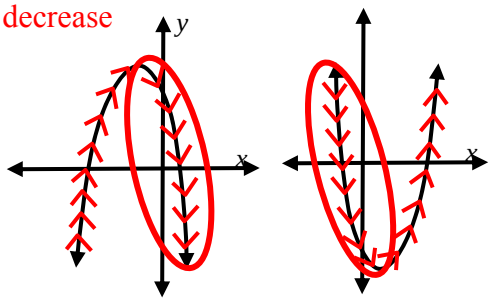
17. Interval of Increase

Domain (x values) where the y values increase



18. Interval of Decrease

Domain (x values) where the y values decrease



19. Translation – slide

$y = a(x-h)^2 + k$

horizontal translation

h > 0 right

h < 0 left

h is the opposite

vertical translation

k > 0 up

k < 0 down

k is NOT the opposite

y = f(x – h) + k

(x, y)→(x + h, y + k)

20. Reflection – flip

$y = a(x-h)^2 + k$

-1 opposite sign vertical reflection

-1 opposite sign horizontal reflection

y = -f(x) or y = f(-x)

(x, y)→(x, -y) or (x, y)→(-x, y)

21. Dilation – size change

$y = a(x-h)^2 + k$

vertical dilation

|a| > 1 narrower, steeper

|a| < 1 shorter, wider

horizontal dilation

y = af(x) or y = f(ax)

(x, y)→(x, ay) or (x, y)→$(\frac{1}{a}x, y)$