M4T1 Solving Quadratic Equations

Name ____________________

1 Polynomial

Many terms.

$$5x^3 - 3x^2 + 2x + \sqrt{6}$$

- no fractional exponents
- no negative exponents
- no variables in the denominator
- no variables under roots

2 Terms

$$3x^2 + 5x + 2$$

After the expression is simplified a term is a product of factors and is separated by addition or subtraction

3 Coefficient, Base, Exponent

$$12x^6$$

Exponent small number

Coefficient Number in front of the variable

Base Number or variable raised to a power

4 Vocabulary

Monomial – one term polynomial
Binomial – two terms polynomial
Trinomial – three terms polynomial

Types of equations or functions

Linear – highest exponent of 1
Quadratic – highest exponent of 2
Cubic – highest exponent of 3
Quartic – highest exponent of 4
Quintic – highest exponent of 5

5 Degree & Leading Coefficient

$$4x^2 + 8x + 3$$

Degree - The highest exponent of a term or the highest sum of the exponents in a term. (degree of 2)

Leading Coefficient – number in front of the term with the highest exponent. (leading coefficient is 4)

6 Adding Polynomials

Only add like terms

ex. $2x + 5x = 7x$

ex. $9x^2 + 5x^3 - 2x^3 + 4x^2 =$
$13x^2 + 3x^3$

7 Multiplying monomials

When multiplying, multiply the coefficients (numbers in front) and add the exponents with the same base.

Ex. $2x \cdot 4x = 8x^2$

Ex. $5x^4 \cdot 2x^2 = 10x^6$

8 Multiplying - distributing

Multiply the factor in front of the parentheses by every term in the parentheses.

$$5x(3x^2 + 2x - 4) =$$
$$15x^3 + 10x^2 - 20x$$

9 Multiplying binomials Distributing

$$(x+2)(x-5) = x(x-5) + 2(x-5)$$
$$= x(x-5) + 2(x-5)$$
$$= x^2 - 5x + 2x - 10$$
$$= x^2 - 3x - 10$$

10 Multiplying binomials Multiplication table

Multiply $(x - 6)(x + 3)$

•	x	- 6
x	x^2	- 6x
+ 3	+3x	- 18

$(x - 6)(x + 3) = x^2 - 3x - 18$

11 Multiplying binomials – F O I L

First Outer Inner Last

F O I L

$$(x+6)(x+7) = x^2 + 7x + 6x + 42$$
$$= x^2 + 13x + 42$$

12 Multiplying – Multiplication Table Expanded

Multiply $(x + 2)(x^2 - 3x + 4)$

•	x^2	-3x	+ 4
x	x^3	$-3x^2$	+4x
+ 2	$+2x^2$	-6x	+8

$(x + 2)(x^2 - 3x + 4) = x^3 - x^2 - 2x + 8$

13 Factoring – GCF

GCF – Greatest Common Factor

Divide out what each term has in common.
Look for common numerical factors.
Look for common variable factors.
Look for common negative signs.

14 Factoring Trinomial – Signs

If the constant is positive the signs in the binomial will be the same, both the sign of the linear term.

If the constant is negative the signs in the binomial will be one negative and one positive.

15 Factoring $x^2 + bx + c$

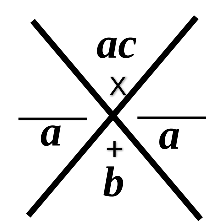
What multiplies to give you "c" but adds to give you b?

Ex. $x^2 - 6x + 8 = (x - 2)(x - 4)$

16 Factoring $ax^2 + bx + c$

X FACTOR

What multiplies to give you *ac* but adds to give you *b*. Divide by ***a***, reduce, bottoms up.



17 Zero Product Property

If ab = 0 then a = 0 or b = 0

If two factors multiply to equal zero, then at least one of the two factors must equal zero.

18 Solving by factoring

1. set equation equal to 0
2. factor the trinomial
3. set factors equal to zero
4. solve the factors

19 Solving by factoring

Example: Solve $x^2 + 2x = 15$

$x^2 + 2x - 15 = 0$

$(x - 3)(x + 5) = 0$

$x - 3 = 0 \quad x + 5 = 0$

$x = 3 \quad x = -5$

20 Difference of two squares

$$a^2 - b^2 = (a-b)(a+b)$$

Two squares subtracting, always factors into the same binomials with different middle signs.

21 Perfect square trinomial

Trinomial that factors into a perfect square.

$$16x^2 - 24x + 9 = (4x - 3)^2$$

Perfect square — Double those square roots. — Perfect square — Square root 1st term — Middle sign — Square root last term

22 Sum of two cubes

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

First parenthesis is the same without the cubes. Second parenthesis, first term squared, opposite of product, second term squared.

23 Difference of two cubes

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

First parenthesis is the same without the cubes. Second parenthesis, first term squared, opposite of product, second term squared.

24 Perfect squares & cubes

$1^2 = 1$	$11^2 = 121$	$1^3 = 1$
$2^2 = 4$	$12^2 = 144$	$2^3 = 8$
$3^2 = 9$	$13^2 = 169$	$3^3 = 27$
$4^2 = 16$	$14^2 = 196$	$4^3 = 64$
$5^2 = 25$	$15^2 = 225$	$5^3 = 125$
$6^2 = 36$	$16^2 = 256$	$6^3 = 216$
$7^2 = 49$	$17^2 = 289$	$7^3 = 343$
$8^2 = 64$	$18^2 = 324$	$8^3 = 512$
$9^2 = 81$	$19^2 = 361$	$9^3 = 729$
$10^2 = 100$	$20^2 = 400$	$10^3 = 1000$

25 Extracting perfect squares

Break the radicand into factors that are perfect squares. Take the square root of the perfect squares.

Examples:

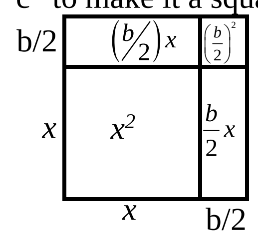
$$\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4} \cdot \sqrt{5}$$
$$= \pm 2\sqrt{5}$$

26 Solving a Perfect Square

1. Isolate the perfect square.
2. Square root both sides of the equation. (don't forget the $\pm$)
3. Solve the two equations.

27 Completing the square diagram

Find "c" to make it a square



$x^2 + bx + ? = (x + \ ? \ )^2$

28 Completing the square

$$x^2 + bx + \underline{\left(\frac{b}{2}\right)^2} = \left(x + \frac{b}{2}\right)^2$$

29 Solving by Completing the square

1. Isolate x^2 and x term
2. Make sure coefficient of x^2 is 1
3. Complete the square.
4. Factor into a perfect square.
5. Square root
6. Solve for x.

30 Solving by Completing the square – Example

$4x^2 + 8x - 32 = 0$

$4x^2 + 8x = 32$

$x^2 + 2x = 8$

$x^2 + 2x + \_\_\_\_ = 8 + \_\_\_\_$

$x^2 + 2x + 1 = 9$

$(x + 1)^2 = 9$

$x + 1 = 9 \quad x + 1 = -9$

$x = 8 \quad x = -10$

31 Quadratic Formula

If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

32 Discriminant

When $ax^2 + bx + c = 0$

then $b^2 - 4ac$ is the discriminant.

If $b^2 - 4ac > 0$, then two real roots or two real zeros. (perfect square – 2 rational, not perfect square – 2 irrational)

If $b^2 - 4ac = 0$, then one real double root or one real zeros. (2 equal rational roots)

If $b^2 - 4ac < 0$, then no real roots or no real zeros. (no real – imaginary)